TEST ID **00038772**

AIEEE PART TEST -1

Hint & Solution

PHYSICS

1. (c)

Here, $X = \frac{Q}{B}$. But $V = \frac{W}{Q}$

$\therefore \quad X = \frac{Q^2}{W}$. Now, $Z = B = \frac{F}{IL}$

$\therefore \quad Y = \frac{X}{3Z^2} = \frac{1}{3}\frac{Q^2}{W} \times \frac{I^2L^2}{F^2}$

$= \frac{A^2T^2A^2L^2}{\left[M\,L^2T^{-2}\right]\left[M^2L^2T^{-4}\right]} = [M^{-3}L^{-2}T^8A^4]$

2. (c)

5.69×10^{15} kg has 3 significant figures as the power of 10 are not considered for significant figures.

3. (b)

$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l}$ (1)

where

M = 1.0 kg (exact), g = 9.8 ms^{-2} (exact)

L = 2 m (exact), l = 0.8 mm = 0.8×10^{-3}m

Δl = ±0.05 mm, d = 0.4 mm = 0.4×10^{-3}m

Δd = ±0.01 mm

Substituting the values of M, g, L, d and l in Eq. (1)

we get, $Y = 2.0\times10^{11}\,\text{Nm}^{-2}$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$ Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2\times0.01\text{mm}}{0.4\text{ mm}} + \frac{0.05\text{ mm}}{0.8\text{ mm}}$$

$$= 0.05 + 0.0625 = 0.1125$$

$$\therefore \quad \Delta Y = 0.1125\times Y = 0.1125\times 2.0\times10^{11}$$

$$= 0.225\times10^{11}\,\text{Nm}^{-2}$$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2\times10^{11}\,\text{Nm}^{-2}$. Therefore, the result of the experiment is

$$Y + \Delta Y = \left(2.0 \pm 0.2\right)\times10^{11}\,\text{Nm}^{-2}$$

4. (a)

The maximum permissible error in η is given by the relation

$$\frac{\Delta\eta}{\eta} = 4\frac{\Delta R}{R} + \frac{\Delta l}{l} + \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$$

It is clear that the error in the measurement of R is magnified four times on account of the occurrence of R^4 in the formula. Hence the radius (R) of the capillary tube must be measured most accurately. Thus the quantity which is raised to the highest power needs the most accurate measurement.

5. (b)

The dimension of the two sides of proportionality are

$$L^3 = L^{2\alpha}(LT^{-1})^{\beta}T^{\gamma} = L^{2\alpha+\beta}T^{\gamma-\beta}$$

Equating the powers of dimensions on both sides, we have

$2\alpha + \beta = 3$

$\gamma - \beta = 0$ which give $\beta = \gamma$ and $\alpha = \frac{1}{2}(3-\beta)$,

i.e., $\alpha \neq \beta = \gamma$.

6. (a)

Let $n \propto l^a \rho^b Y^c$

Putting dimensions of all the quantities, we have

$$\left(T^{-1}\right) \propto L^a(ML^{-3})^b(ML^{-1}T^{-2})^c$$

Equating powers of M, L and T on both sides, we get $b + c = 0$, $a - 3b - c = 0$ and $-2c = -1$

which give $a = -1, b = -\frac{1}{2}$ and $c = \frac{1}{2}$. Thus $n \propto l^{-1} \rho^{1/2} Y^{1/2}$.

7. (a)

8. (d)

Energy density = $\frac{energy}{volume}$ = $[ML^{-1}T^{-2}]$

Force/area = $ML^{-1}T^{-2}$

[charge/volume] × [voltage] = $\frac{Q}{vol.}, \frac{W}{Q}$

$= \frac{work}{volume} = ML^{-1}T^{-2}$

The dimensions of (4) are different,

i.e., $\frac{ML^2T^{-1}}{M} = M^0L^2T^{-1}$

Hence, correct choice is (d)

9. (d)

$$[K] = \left[\frac{\text{Volume } (y_2 - y_1)}{\text{Time } (x_2 - x_1)^4}\right]$$

$$= \frac{L^3 \cdot L}{T \cdot L^4} = T^{-1}M^0L^0$$

10. (d)

$$n = -\frac{D(n_2 - n_1)}{x_2 - x_1} \Rightarrow T^{-1}L^{-2} = \frac{D(L^{-3})}{L}$$

$$\Rightarrow D = \frac{T^{-1}L^{-2} \times L}{L^{-3}} \Rightarrow D = [M^0L^2T^{-1}]$$

11. (c)

Used $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix} = -4\hat{i} + 8\hat{j}$

12. (a)

The position vector is given by

$\vec{r} = v_0 t\hat{i} + a \sin \omega t \hat{j}$

Here $\vec{r}_0 = \vec{O}$

and, $\vec{r} = v_0 t\hat{i} + a \sin \omega t\, \hat{j}$

$$\Rightarrow at\ t = \frac{2\pi}{2\omega}, \vec{r} = v_0\left(\frac{3\pi}{2\omega}\right)\hat{i} + a \sin \omega\left(\frac{3\pi}{2\omega}\right)\hat{j}$$

$$= v_0\left(\frac{3\pi}{2\omega}\right)\hat{i} - a\hat{j}$$

Distance from origin is $|\vec{r}| = \sqrt{a^2 + \left(\frac{3\pi v_0}{2\omega}\right)^2}$

13. (a)

The resultant R of vectors (A + B) and (A – B) is

R = (A + B) + (A – B) = 2A

∴ The magnitude of the resultant = 2A

14. (b)

The angle between two vectors of equal magnitude is 120^o. So, resultant has the same magnitude as either of the given vectors. Moreover, it is mid-way between the two vectors, i.e., it is along x-axis.

15. (d)

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{r} = (4\cos 10t)\hat{i} + (3\sin 10t)\hat{j}$

$\vec{r}\left(t = \frac{\pi}{20}\right) = 3\hat{j}$

also $\vec{v} = (-40\sin 10t)\hat{i} + (30\cos 10t)\hat{j}$

$\vec{v}\left(t = \frac{\pi}{20}\right) = -40\hat{i}$

$\vec{v} \cdot \vec{r} = 0$, also $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = -100\vec{r}$

16. (c)

Time of flight

$T = \sqrt{\frac{2H}{g}} \Rightarrow T \propto \sqrt{H}$

17. (b)

Given $7x = \frac{g}{2}(2n-1)$ and $x = \frac{1}{2}g(1)^2$

Solving equation we get n = 4s.

18. (d)

$v = \left(\sqrt{4+4s}\right)m/s$

$\frac{ds}{dt} = \sqrt{4+4s}$

$\frac{ds}{(4+4s)^{1/2}} = dt$

$\int_0^s (4+4s)^{1/2} ds = \int_0^2 dt$

Integrating both side-

$$\left[\frac{(4+4s)^{1/2}}{\frac{1}{2}.4}\right]_0^s = [t]_0^2$$

$$\frac{(4+4s)^{1/2}}{2} - 1 = 2$$

$$\frac{(4+4s)^{1/2}}{2}=3$$

$$(4+4s)^{1/2}=6$$

Squaring both side

4 + 4s = 36

4s = 32

s = 8m

19. (a)

$$h = \frac{1}{2}gT^2 \Rightarrow T^2 = \frac{2h}{g}$$

Let x be the position of the ball from the ground at $t = \frac{T}{3}$

$$\therefore h-x=\frac{1}{2}g\left(\frac{T}{3}\right)^2 \ \therefore x=h-\frac{1}{2}g\frac{2h}{9g}=\frac{8}{9}h$$

20. (b)

As x-t graph is a parabola, 1st velocity will decrease becomes zero and increases in opposite direction.

21. (c)

$$V=\frac{4}{3}\pi r^3$$

Differentiate with respect to t,

$$\frac{dV}{dt}=\frac{4}{3}\pi 3r^2.\frac{dr}{dt}\Rightarrow\frac{dr}{dt}=\frac{1}{4\pi r^2}.\frac{dV}{dt}$$

$$\frac{dr}{dt}=\frac{1}{4\times\pi\times15\times15}\times900\Rightarrow\frac{dr}{dt}=\frac{1}{\pi}=\frac{7}{22}.$$

22. (a)

Time taken by the body to strike the inclined plane

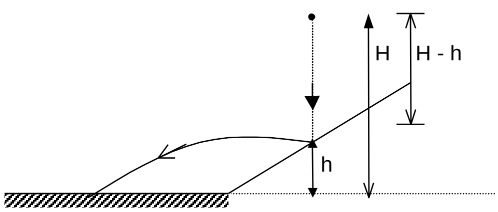
$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

Now as after impact the velocity of the body is horizontal, so time taken to reach the ground

$$t_2 = \sqrt{\frac{2h}{g}}$$

So total time of motion -

$$t = t_1 + t_2 = \sqrt{\frac{2}{g}}\left[\sqrt{h}+\sqrt{H-h}\right]$$



23. (a)

r = 1 km = 1000 m,

v = 900 km/h = 9000 × 5/18 = 250 m/s

$$a_c=\frac{v^2}{r}=\frac{(250)^2}{1000}=62.5m/s^2$$

24. (c)

Let the vectors be

$\vec{A}$ & $\vec{B}$

$|\vec{A}| = |\vec{B}|$ (given)

$\frac{R_2}{R_1} = \sqrt{3}$ (given)

$$\Rightarrow \frac{R_2}{R_1}$$

$$=\frac{\sqrt{A^2+B^2-2AB\cos\theta}}{\sqrt{A^2+B^2+2AB\cos\theta}}=\sqrt{3}$$

Putting A = B we obtain θ = 120°.

25. (d)

$$\left|\vec{v_f}-\vec{v_i}\right|=\sqrt{v_0^2+v_0^2-2v_0^2\cos\theta}$$

$$=v_0^2\sqrt{2(1-\cos\theta)}$$

$$=v_0^2\sqrt{2\times2\sin^2(\theta/2)}=2v_0\sin(\theta/2)$$

26. (a)

$v_1 = \sqrt{4gH}$ $\qquad v_2 = \sqrt{16gH}$

$t_1 = \sqrt{\frac{H}{g}}$, $t_2 = \sqrt{\frac{H}{4g}}$

$t = t_1 - t_2$

$$\sqrt{\frac{H}{g}}-\sqrt{\frac{H}{4g}}=t \qquad \Rightarrow \frac{1}{2}\sqrt{\frac{H}{g}}=t$$

$$H=4t^2g$$

$$v_2-v_1=v\Rightarrow v=2\sqrt{4gH}-\sqrt{4gH}$$

$$v=2\sqrt{gH}-2\sqrt{4t^2g^2}$$

v = 4gt

27. (b)

$$h = (u\sin\theta)t - \frac{1}{2}gt^2$$

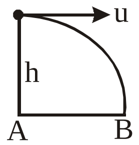
$$d = (u\cos\theta)t \text{ or } t = \frac{d}{u\cos\theta}$$

$$h = u\sin\theta\times\frac{d}{u\cos\theta}-\frac{1}{2}g\times\frac{d^2}{u^2\cos^2\theta}$$

$$u=\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta-h)}}$$

28. (c)

$AB = u\sqrt{\frac{2h}{g}}$



$= 600 \times \frac{5}{18}\sqrt{\frac{2\times 2000}{10}}$ = 3333 m = 3.33 km

29. (c)

Let $y=\sqrt{x^2+16}$ and $z=\frac{x}{x-1}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{2}(x^2+16)^{-1/2}(2x)$

& $\frac{dz}{dx}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$\therefore \frac{dy}{dz}=\frac{-x}{\sqrt{x^2+16}}\frac{1}{1/(x-1)^2}$

$\left(\frac{dy}{dz}\right)_{x=3}=\frac{-3(2)^2}{5}=\frac{-12}{5}$

30. (a)

$\int(\sin^{-1}x+\cos^{-1}x)dx=\int\left(\frac{\pi}{2}\right)dx=\frac{\pi x}{2}+c$

$\left(\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right)$

MATHEMATICS

31. (b)

$A=[x:x\in R,-1<x<1]$

$B=[x:x\in R:x-1\le -1 \text{ or } x-1\ge 1]$

$=[x:x\in R:x\le 0 \text{ or } x\ge 2]$

$\therefore A\cup B=R-D$

where $D=[x:x\in R, 1\le x<2]$

32. (d)

$\cos 2x=1-2\sin^2 x$ and put $2^{-\sin^2 x}=t$

$\Rightarrow 2^{\cos 2x}=2^{1-2\sin^2 x}=2(2^{-\sin^2 x})^2=2t^2$

$\Rightarrow 2t^2-3t+1=0$

$\Rightarrow t=1, 1/2$

$\Rightarrow 2^{-\sin^2 x}=1=2^o$

$\Rightarrow \sin^2 x=0$ or $x=n\pi,\ n\in Z$

from $2^{-\sin^2 x}=\frac{1}{2}=2^{-1}$

$\Rightarrow \sin x=1$ or $x=n\pi\pm\frac{\pi}{2}, n\in Z$

33. (c)

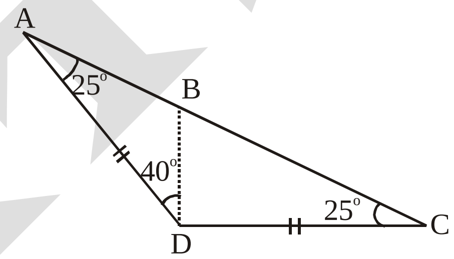
$\angle BDC = 90^o$, $\angle DAC = 180^o - (130^o + 25^o) = 25^o$

AD = DC = BD cot25^o

$=2\frac{\sqrt{1-\sin^2 25^o}}{\sin 25^o}$

$=2\frac{\sqrt{1-(0.423)^2}}{0.423}$

= 4.28 km



34. (a)

$R=\{(x,y):4x+3y=20\}$

[$\because$ 2, 4 $\in$ N and 4(2) + 3(4) = 8 + 12 = 20]

35. (a)

$f(x)$ is defined if $\log x\ne 0$ and $x>0\Rightarrow x\ne 1$ and $x>0$

$\therefore$ domain of

$f=(0,1)\cup(1,\infty)=(0,\infty)-\{1\}$.

36. (a)

$\sin\theta=\frac{1}{2}$ and $\cos\theta=-\frac{\sqrt{3}}{2}$

$\Rightarrow$ θ lies in the second quadrant.

$\Rightarrow \sin\theta=\sin\frac{5\pi}{6};\cos\theta=\cos\frac{5\pi}{6}$

$\therefore \theta=2n\pi+\frac{5\pi}{6}$

37. (c)

$A-B=\frac{\pi}{4}\Rightarrow \tan(A-B)=\tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan A-\tan B}{1+\tan A\tan B}$

$\Rightarrow \tan A-\tan B-\tan A\tan B=1$

$\Rightarrow \tan A-\tan B-\tan A\tan B+1=2$

$\Rightarrow (1+\tan A)(1-\tan B)=2$

$\Rightarrow y=2$

Hence, $(y+1)^{y+1}=(2+1)^{2+1}=(3)^3=27$

38. (b)
A < B < C < D is in an ascending order ….(1)
sinA = sinB = sinC = sinD = k(+ ve)....(2)
We have chosen the values to satisfy the condition of equations (1) and (2)

$$E=4\sin\frac{A}{2}+3\sin\frac{\pi-A}{2}+2\sin\frac{2\pi+A}{2}+\sin\frac{3\pi-A}{2}$$

$$=4\sin\frac{A}{2}+3\cos\frac{A}{2}-2\sin\frac{A}{2}-\cos\frac{A}{2}$$

$$=2\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)$$

$$=2\sqrt{\sin^2\frac{A}{2}+\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$=2\sqrt{1+\sin A}=2\sqrt{1+k}$$

Hence, (b) is the correct answer.

39. (a)

$$(A\times B)\cap(B\times A)=(A\cap B)\times(B\cap A)$$

$$=(2,3)\times(2,3)$$

$$=\{(2,2),(2,3),(3,2),(3,3)\}$$

$\therefore$ number of elements = 4

40. (b)

$$A=\{x:|x|<3, x\in Z\}$$

$$=\{x:-3<x<3, x\in Z\}=\{-2,-1,0,1,2\}$$

$$R=\{(x,y): y=|x|, x\neq -1\}$$

$$=\{(-2,2),(0,0)(1,1),(2,2)\}$$

Since R has 4 elements

$\therefore$ Number of elements in the power ser of $R=2^4=16$.

41. (b)

$$\Rightarrow 2\cos\theta\left(-\frac{1}{2}+2\cos^2\theta-1\right)=1$$

$$\Rightarrow 4\cos^3\theta-3\cos\theta=1$$

$$\Rightarrow \cos 3\theta=1=\cos 0$$

$$\Rightarrow 3\theta=2n\pi \text{ or } \theta=\frac{2n\pi}{3}, n\in Z$$

Given the values so the $2n$ does not exceed 18.

$\therefore n=0,1,2,3,......9$

Hence the sum

$$=\frac{2\pi}{3}\sum_{1}^{9}n=\frac{2\pi}{3}\times\frac{9(9+1)}{2}=30\pi$$

42. (b)
Given, sin α + sin β = a
And cos α + cos β = b
Now
$(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2=b^2+a^2$
$\Rightarrow \cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta+\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta=b^2+a^2$
$\Rightarrow (\cos^2\alpha+\sin^2\alpha)+(\cos^2\beta+\sin^2\beta)+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=a^2+b^2$
$\Rightarrow 2+2\cos(\alpha-\beta)=a^2+b^2$

or, $\cos(\alpha-\beta)=\dfrac{a^2+b^2-2}{2}$

Now, $\tan\dfrac{\alpha-\beta}{2}=\pm\sqrt{\dfrac{1-\cos(\alpha-\beta)}{1+\cos(\alpha-\beta)}}$

$$=4\pm\sqrt{\frac{1-\frac{a^2+b^2-2}{2}}{1+\frac{a^2+b^2-2}{2}}}=\pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

43. (d)

$$n(M)=23, n(P)=24, n(C)=19$$

$$n(M\cap P)=12, n(M\cap C)=9,$$

$$n(P\cap C)=7$$

$$m(M\cap P\cap C)=4.$$

We have to find

$$n(M\cap P'\cap C'), n(P\cap M'\cap C'),$$

$$n(C\cap M'\cap P')$$

Now

$$n(M\cap P'\cap C')=n\left[M\cap(P\cap C)'\right]$$

$$=n(M)-n(M\cap(P\cap C))$$

$$=n(M)-n[(M\cap P)\cup(M\cap C)]$$

$$=n(M)-n\left[\begin{array}{r}(M\cap P)\cup(M\cap C)+\\ n(M\cap P\cap C)\end{array}\right]$$

$$=23-12-9+4=27-21=6$$

$n(P\cap M'\cap C') = n\left[P\cap(M\cup C)'\right]$

$= n(P) - n\left[P\cap(M\cup C)\right]$

$= n(P) - n\left[(P\cap M)\cup(P\cap C)\right]$

$= n(P) - n(P\cap M) - n(P\cap C) +$
$n(P\cap M\cap C)$

$= 24 - 12 - 7 + 4 = 9$

$n(C\cap M'\cap P') = n(C) - n(C\cap P)$
$- n(C\cap M) + n(C\cap P\cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

44. (b)

$n\left((A\times B)\cap(B\times A)\right)$

$= n\left((A\cap B)\times(B\cap A)\right)$

$= n(A\cap B).n(B\cap A)$

$= n(A\cap B).n(A\cap B)$

$= (99)(99) = 99^2$

45. (a)

Let, $y = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$

$\Rightarrow \left(\alpha + 6x - 8x^2\right)y = \alpha x^2 + 6x - 8$

$\Rightarrow (\alpha + 8y)x^2 + 6(1-y)x - (8 + \alpha y) = 0$

Since x is real.

$\therefore 36(1-y)^2 + 4(\alpha + 8y)(8 + 2y) \ge 0$

$\Rightarrow 9\left(1 - 2y + y^2\right) + \left(8\alpha + \alpha^2 y + 64y + 8\alpha y^2\right) \ge 0$

$\Rightarrow y^2(9 + 8\alpha) + y\left(\alpha^2 + 46\right) + (9 + 8\alpha) \ge 0$ .....(1)

(1) will hold for each $y \in R$ if $9 + 8\alpha > 0$

and $\left(46 + \alpha^2\right)^2 - 4(9 + 8\alpha)^2 \le 0$ (Disc. ≤ 0)

$\Rightarrow \alpha > -\dfrac{9}{8}$ and $\left[46 + \alpha^2 - (9 + 8\alpha)\right]$
$\left[46 + \alpha^2 + 2(9 + 8\alpha)\right] \le 0$

$\Rightarrow \alpha > -\dfrac{9}{8}$ and $\left[\alpha^2 - 16\alpha + 28\right]$
$\left[\alpha^2 - 16\alpha + 64)\right] \le 0$

$\Rightarrow \alpha > -\dfrac{9}{8}$ and $(\alpha - 2)(\alpha - 14)(\alpha + 8)^2 \le 0$

$\Rightarrow \alpha > -\dfrac{9}{8}$ and $(\alpha - 2)(\alpha - 14) \le 0$

$(\because (\alpha + 8)^2 \ge 0)$

$\Rightarrow \alpha > -\dfrac{9}{8}$ and $2 \le \alpha \le 14$ $\quad \therefore 2 \le \alpha \le 14$

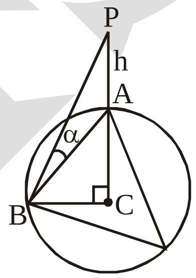
Hence $f(x)$ will be onto if $2 \le \alpha \le 14$

$\therefore$ reqd. interval is [2, 14]

46. (c)

Let AP be the lamp post of height h at a point A on a circular path of radius r and centre C. Let B be the point on the path such that ∠PBA = α.
⇒ AB = h cot α.

Since AB subtends an angle 45° at another point of the path, it subtends an angle of 90° at the centre C so that ∠BCA = 90°



Also CA = CB = r ⇒ AB = $\sqrt{2}$ r and then

h cot $\alpha = \sqrt{2}$r

⇒ h = $\sqrt{2}$ r tan α.

47. (d)

$\tan(\alpha + 2\beta) = \dfrac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta} = \dfrac{\frac{1}{7} + \tan 2\beta}{1 - \frac{1}{7}\tan 2\beta}$

Now, $\tan 2\beta = \dfrac{2\tan\beta}{1 - \tan^2\beta} = \dfrac{2\times\frac{1}{3}}{1 - \frac{1}{9}} = \dfrac{3}{4}$

$[\tan\beta > 0\ as\ 0 < \beta < \pi/2]$

Substituting the value of tan 2β in (i), we get, tan

$(\alpha + 2\beta) = \dfrac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}\times\frac{3}{4}} = \dfrac{25}{25} = 1$

Now, $0 < \alpha < \dfrac{\pi}{2}$ and $0 < \beta < \dfrac{\pi}{2}$

$\therefore 0 < 2\beta < \pi,$ but $\tan 2\beta = \dfrac{3}{4} > 0$

$\Rightarrow 0 < 2\beta < \dfrac{\pi}{2}$

Hence, 0 < α + 2β < π

In the interval (0, π), tan θ takes value 1 at $\dfrac{\pi}{4}$ only

$\therefore \alpha + 2\beta = \dfrac{\pi}{4}$

48. (b)

Let $f(T+x) = f(x)$

$\Rightarrow T + x - [T+x] = x - [x]$

$\Rightarrow T = [T+x] - [x] =$ An integer.

Clearly the least positive value of T independent of $x = 1$

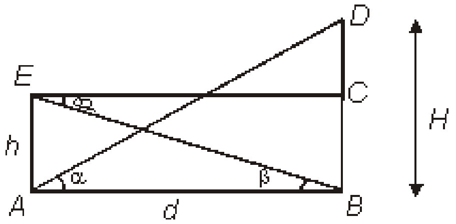
Hence $f(x)$ is a periodic function of period 1.

49. (a)

Clearly $A = \{2,3\}, B = \{2,4\}, C = \{4,5\}$

$B \cap C = \{4\} \therefore A \times (B \cap C) = \{(2,4),(3,4)\}$

50. (b)



Let BD = H

AB = d

ΔABD

$\tan\alpha = \dfrac{H}{d}$

d = H cot α........(i)

ΔABE

$\tan\beta = \dfrac{h}{d}$ $\quad \tan\beta = \dfrac{h}{H\cot\alpha}$

H = h tanα.cotβ

51. (d)

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 12 + 9 - 4 = 17$

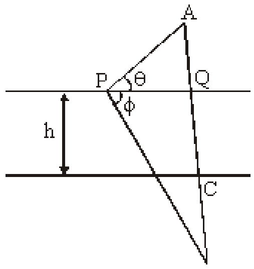
Now $n\left((A \cup B)^C\right) = n(U) - n(A \cup B)$

$= 20 - 17 = 3$

52. (c)

AC = BC = x

then AQ = x – h and BQ = x + h



Now, PQ = (x – h) cot θ = (x + h) cot φ

$\Rightarrow x = h\dfrac{\cot\theta + \cot\phi}{\cot\theta - \cot\phi} = h\dfrac{\sin(\theta+\phi)}{\sin(\theta-\phi)}$

53. (d)

For D_f, $x - 1 > 0$ *i.e.* $x > 1$ *i.e.* $x \in (1, \infty)$ and

$2x - 3 > 0$ *i.e.* $x > \dfrac{3}{2}$ *i.e.* $x \in \left(\dfrac{3}{2}, \infty\right)$

$\therefore x \in \left(\dfrac{3}{2}, \infty\right)$

Also $5 - 2x \ge 0 \Rightarrow 5 \ge 2x \Rightarrow x \le \dfrac{5}{2}$

$\Rightarrow x \in \left(-\infty, \dfrac{5}{2}\right] \therefore D_f$ is $\left(\dfrac{3}{2}, \dfrac{5}{2}\right]$

54. (b)

It is given that α and β are distinct roots of a cos θ + b sin θ = c

∴ a cos α + b sin α = c and a cos β + b sin β = c

⇒ (a cosα + b sinα)–(a cosβ + b sinβ) c – c

⇒ a (cos α – cos β) + b (sin α – sin β) = 0

$\Rightarrow -2a\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2} = 2b\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

$\Rightarrow \tan\dfrac{\alpha+\beta}{2} = \dfrac{b}{a}$

Now, $\sin(\alpha+\beta) = \dfrac{2\tan\dfrac{\alpha+\beta}{2}}{1+\tan^2\dfrac{\alpha+\beta}{2}}$

$\Rightarrow \sin(\alpha+\beta) = \dfrac{2\dfrac{b}{a}}{1+\dfrac{b^2}{a^2}} = \dfrac{2ab}{a^2+b^2}$

55. (c)

Since $2^m - 2^n = 48 = 16 \times 3 = 2^4 \times 3$

$= 2^n\left(2^{m-n} - 1\right) = 2^4 \times 3$

$\therefore n = 4$ and $2^{m-n} = 4 = 2^2$

$\Rightarrow m - n = 2 \Rightarrow m - 4 = 2 \Rightarrow m = 6$

$\therefore m = 6, n = 4$

56. (d)

$|2+3i| = \sqrt{(2)^2 + (3)^2} = \sqrt{13} \neq 13$

$\therefore (2+3i)R\ 13$ is wrong.

$|3| = \sqrt{(3)^2 + 0^2} = \sqrt{9} = 3 \neq -3$

$\therefore 3R(-3)$ is wrong

$|1+i| = \sqrt{(1)^2 + (1)^2} = \sqrt{2} \neq 2$

$\therefore$ $(1 + i)R^2$ is wrong

$|i| = \sqrt{0^2 + 1^2} = 1$

$\therefore iR1$ is correct.

57. (a)

We have,

$\tan(x + 100^o) = \tan(x + 50^o)\tan x \tan(x - 50^o)$

$\Rightarrow \dfrac{\tan(x+100^o)}{\tan(x-50^o)} = \tan(x+50^o)\tan x$

$\Rightarrow \dfrac{\sin(x+100^o)\cos(x-50^o)}{\cos(x+100^o)\sin(x-50^o)} = \dfrac{\sin(x+50^o)\sin x}{\cos(x+50^o)\cos x}$

$\Rightarrow \dfrac{\sin(x+100^o\cos(x-50^o)+\cos(x+100^o)\sin(x-50^o)}{\sin(x+100^o)\cos(x-50^o)-\cos(x+100^o)\sin(x-50^o)}$

$= \dfrac{\sin(x+50^o)\sin x + \cos(x+50^o)\cos x}{\sin(x+50^o)\sin x - \cos(x+50^o)\cos x}$

$\Rightarrow \dfrac{\sin(x+100^o + x - 50^o)}{\sin(x+100^o - x + 50^o)}$

$= \dfrac{\cos(x+50^o - x)}{-\cos(x+50^o + x)}$

$\Rightarrow \dfrac{\sin(2x+50^o)}{\sin 150^o} = \dfrac{\cos 50^o}{-\cos(2x+50^o)}$

$\Rightarrow$ $\sin(2x + 50^o)\cos(2x + 50^o)$

$= -\sin 150^o \cos 50^o$

$\Rightarrow$ $2\sin(2x + 50^o)\cos(2x + 50^o) = -2\cos 60^o$

$\cos 50^o$ $[\because \sin 150^o = \cos 60^o]$

$\Rightarrow$ $\sin(4x + 100^o) = \sin(270^o - 50^o)$

$\Rightarrow$ $\sin(4x + 100^o) = \sin 220^o$

$\Rightarrow$ $4x + 100^o = 220^o$

$\Rightarrow$ $x = 30^o$

58. (a)

$\sin^6 x + \cos^6 x = \dfrac{1}{8}[(1-\cos 2x)^3 + (1+\cos 2x)^3]$

$= \dfrac{1}{4}[1 + 3\cos^2 2x] = a^2$

$\Rightarrow \cos^2 2x = \dfrac{1}{3}[4a^2 - 1]$

$\cos^2 2x \leq 1$

$\Rightarrow a^2 \leq 1 \Rightarrow a \in (-1, 1)$

59. (c)

We have, $\tan\alpha = \dfrac{p}{q}$

$\therefore \sin\alpha = \dfrac{p}{\sqrt{p^2+q^2}}$ and $\cos\alpha = \dfrac{q}{\sqrt{p^2+q^2}}$

$= \dfrac{1}{2}\{p\,\mathrm{cosec}\ 2\beta - q\sec\ 2\beta\}$

$= \dfrac{\sqrt{p^2+q^2}}{2}\left\{\dfrac{p}{\sqrt{p^2+q^2}}\mathrm{cosec}\ 2\beta - \dfrac{q}{\sqrt{p^2+q^2}}\sec 2\beta\right\}$

$= \dfrac{\sqrt{p^2+q^2}}{2}\left\{\dfrac{\sin\alpha}{\sin 2\beta} - \dfrac{\cos\alpha}{\cos 2\beta}\right\}$

$= \dfrac{\sqrt{p^2+q^2}}{2}\left\{\dfrac{\sin\alpha\cos 2\beta - \cos\alpha\sin 2\beta}{\sin 2\beta\cos 2\beta}\right\}$

$= \sqrt{p^2+q^2}\left\{\dfrac{\sin(\alpha - 2\beta)}{2\sin 2\beta\cos 2\beta}\right\}$

$= \sqrt{p^2+q^2}\left\{\dfrac{\sin(6\beta - 2\beta)}{\sin 4\beta}\right\}$ $[\because \alpha = 6\beta]$

$= \sqrt{p^2+q^2}$

60. (d)

When $-\dfrac{\pi}{4} < x < 0$

$\Rightarrow -1 < -\dfrac{\pi}{4} < x < 0 \Rightarrow [x] = -1$

$\Rightarrow y = \sin(-1) = -\sin 1$

when $0 \leq x < \dfrac{\pi}{4} < 1 \Rightarrow [x] = 0$

$\Rightarrow y = \sin 0 = 0 \therefore R_f = \{0, -\sin 1\}$.

CHEMISTRY

61. (c)
Let the weight of oxygen = x
$\therefore$ The weight of nitrogen = $4x$
Number of molecules of oxygen $= \frac{x}{32} \times N_0$
Number of molecules of nitrogen $= \frac{4x}{28} \times N_0$
$$\frac{\text{number of molecules of oxygen}}{\text{number of molecules of nitrogen}} = \frac{x \times 28}{32 \times 4x}$$
$$= \frac{7}{32}$$

62. (a)
Number of molecules in 36 g of water
$$= \frac{36}{18} \times 6.023 \times 10^{23}$$
$$= 2 \times 6.023 \times 10^{23}$$
Number of molecules in 28 g of CO
$$= \frac{28}{28} \times 6.023 \times 10^{23}$$
$$= 1 \times 6.023 \times 10^{23}$$
Number of molecules in 46 g of C_2H_5OH
$$= \frac{46}{46} \times 6.023 \times 10^{23}$$
$$= 1 \times 6.023 \times 10^{23}$$
Number of molecules in 54 g of N_2O_5
$$= \frac{54}{108} \times 6.023 \times 10^{23}$$
$$= 0.5 \times 6.023 \times 10^{23}$$
Hence, in these 36 g of water has largest number of molecules.

63. (a)
$$\underset{\substack{2[216+12+48] \\ 552\text{g}}}{2Ag_2CO_3} \rightarrow \underset{\substack{\text{residue} \\ 432\text{g}}}{4Ag} + 2CO_2 \uparrow + O_2 \uparrow$$
$\because$ 552 g silver carbonate on strong heating gives = 432 g residue
$\because$ 2.76 g silver carbonate on strong heating yield $= \frac{432 \times 2.76}{552}$ g residue = 2.16 g

64. (c)
$$[Ca \rightarrow Ca^{2+} + 2e^-] \times 3$$
$$[Al^{3+} + 3e^- \rightarrow Al] \times 2$$
$$3Ca \rightarrow 3Ca^{2+} + 6e^-$$
$$2Al^{3+} + 6e^- \rightarrow 2Al$$
$$3Ca + 2Al^{3+} \rightarrow 3Ca^{2+} + 2Al$$
Therefore, the stoichiometric coefficient of Ca in the given reaction is 3.

65. (b)
Equivalent weight $= \frac{\text{molecular weight}}{\text{valency factor}}$
If valency factor is 2, then equivalent weight will be equal to its molecular weight
In $MnSO_4$, the oxidation state of Mn is +II
In Mn_2O_3, the oxidation state of Mn is +III
In MnO_2, the oxidation state of Mn is +IV
In MnO_4^-, the oxidation state of Mn is + VII
In MnO_4^{2-}, the oxidation state of Mn is +VI
Thus, when $MnSO_4$ is converted into MnO_2, then the valency factor is 2, and the equivalent weight of $MnSO_4$ will be half of its molecular weight.

66. (c)
$$\underset{2(36+1+31+64)=264}{2(NH_4)_2HPO_4} \rightarrow \underset{62+80=142}{P_2O_5}$$
$$\%\text{ of } P_2O_5 = \frac{\text{wt. of } P_2O_5}{\text{wt of salt}} \times 100$$
$$= \frac{142}{264} \times 100 = 53.78\%$$

67. (d)
$2KI + HgCl_2 \rightarrow KCl + HgI_2$
$2KI + HgI_2 \rightarrow K_2HgI_4$
For producing 1 mol of K_2HgI_4, KI required = 4 mol
For producing 0.4 mol of K_2HgI4,
KI required = 4 × 0.4 = 1.6 mol

68. (b)
In this reaction
$Cr_2O_7^{2-}$ is changed into Cr^{3+} ion.
So, equivalent weight of
$$K_2Cr_2O_7 = \frac{\text{molecular wt.}}{\text{decrease of oxidation number} \times n}$$
(where n = no. of atoms of Cr)
$$= \frac{\text{molecular wt.}}{3 \times 2} = \frac{\text{molecular wt.}}{6}$$

69. (a)

Strength of oxalic acid $\begin{pmatrix} COOH \\ | \\ COOH \end{pmatrix} 2H_2O$

= 6.3 g/250 mL or strength of oxalic acid = 6.3 × 4 g/L = 25.2 g/L

Normality of oxalic acid solution

$$= \frac{\text{strength in g / L}}{\text{equivalent wt.}} = \frac{25.2}{63} N = 0.4\ N$$

Suppose, x mL volume of 0.1 N NaOH is required to complete neutralization of 10 mL of this solution.

So, meq of NaOH = 0.1 × (x) (by, N × V in mL)

meq of oxalic acid = 0.4 × 10

On complete neutralization, one meq of acid is neutralized with one meq of base.

So, meq of NaOH = meq of oxalic acid

\ 0.1 × (x) = 0.4 × 10

$$x = \frac{0.4' \ 10}{0.1} = 40\,\text{mL}$$

70. (a)

Mixture X contains 0.02 moles of $[Co(NH_3)_5SO_4]Br$ and 0.02 moles of $[Co(NH_3)_5Br]SO_4$ was prepared in two litres of solution. So, the concentration of $[Co(NH_3)_5SO_4]Br$ and $[Co(NH_3)_5Br]SO_4$ in solution are 0.01 mol/L and 0.01 mol/L respectively. During the reaction with $AgNO_3$(excess) AgBr is precipitated as follows

$$\underset{0.01\,mol/L}{[Co(NH_3)_5SO_4]Br} + \underset{(excess)}{AgNO_3} \rightarrow$$

$$\underset{Soluble}{[Co(NH_3)_5SO_4]NO_3} + \underset{\substack{0.01 mol/L \\ (Y)}}{AgBr\downarrow}$$

Hence, number of moles of Y = 0.01

On addition of excess $BaCl_2$, SO_4^{2-} ion of $[Co(NH_3)_5Br]SO_4$ is precipitated in form of Z.

$$\underset{0.01\,mol/L}{[Co(NH_3)_5Br]SO_4} + \underset{(excess)}{BaCl_2} \rightarrow$$

$$\underset{\substack{0.01 mol/L \\ 'Z'}}{BaSO_4\downarrow} + \underset{Soluble}{[Co(NH_3)_5Br]Cl_2}$$

Hence, number of moles of Z = 0.01

Thus, the number of moles of Y and Z are 0.01 and 0.01 respectively.

71. (d)

$$6Fe^{2+} + Cr_2O_7^{2-} + 14H^+ \rightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$$

$$\overset{+6}{Cr_2}O_7^{2-} \rightarrow Cr^{3+}$$

x factor = 6

Mohr's salt, $\overset{+2}{Fe}SO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$

72. (a)

(1) 1 gm of C_2H_4 contains atoms

$$= \frac{8N_A}{23} = 0.28\,N_A$$

(2) 1 g of O_3 contain atoms

$$= \frac{3N_A}{48} = 0.062N_A$$

(3) 1 g of oxygen contains atoms

$$= \frac{2N_A}{32} = 0.062N_A$$

(4) 1 g of He contains atom $= \frac{N_A}{4} = 0.25N_A$.

73. (d)

Moles of NaCl = 3 × 300 × 10^{-3}

∴ Moles of Cl^- ions

= 3 × 300 × 10^{-3} = 0.9 mol

Moles of $BaCl_2$ = 4 × 200 × 10^{-3}

∴ Moles of Cl^- ions

= 2 × 4 × 200 × 10^{-3} = 1.6 mol

Total moles of Cl^- ions in solution

= 0.9 + 1.6 = 2.5 mol

Total volume of solution = 500 ml

∴ Conc. of Cl^- ions $= \frac{2.5 \times 1000}{500} = 5M$

74. (c)

MCl_x + x $AgNO_3$ ® x AgCl + $M(NO_3)x$

Moles of $AgNO_3$ required for precipitation = 500 × 0.8 × 10^{-3}

Now 0.1 mol of MCl_x requires $AgNO_3$ =

0.4 mol

1 mol of MCl_x requires $AgNO_3$ = 4 mol

Hence x = 4

75. (a)

$$4Fe + 3O_2 \rightarrow 2Fe_2O_3$$

$$4 \times 56\,g \qquad 2\,mol(2 \times 160\,g)$$

Amount of iron required for producing 1 mol of Fe_2O_3 = 2 × 56 = 112 g

Heat produced by burning 112 g of iron

$$= \frac{29.28 \times 112}{4.0} = 819.84\,kJ.$$

76. (b)

Here P_1 = 1 atm P_2 = ?

V_1 = 100 litre V_2 = 5 litre

T_1 = 273 K T_2 = 273 K

As temperature is constant

Then from Boyle's law

$P_1V_1 = P_2V_2$
or $1 \times 100 = P_2 \times 5$
$\therefore P_2 = 20$ atm
$\therefore$ Additional pressure that should be applied = $P_2 - P_1 = 20 - 1 = 19$ atm

77. (b)
Density of a gas is given $\rho = \frac{PM}{RT}$.
Obviously the choice that has greater $\frac{P}{T}$ would have greater density.

78. (a)
Weight of balloon = 100 kg $= 10 \times 10^4$ g
Volume of balloon
$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{20}{2} \times 100\right)^3$$
$$= 4190 \times 10^3 \text{ L}$$
Weight of gas (He) in balloon
$$= \frac{PV_m}{RT} = \frac{1 \times 4190 \times 10^3 \times 4}{0.082 \times 300} = 68.13 \times 10^4 \text{ g}$$
$\therefore$ Total weight of gas and balloon
$$= 68.13 \times 10^4 + 10 \times 10^4 = 78.13 \times 10^4 \text{ g}$$
Weight of air displaced
$$= \frac{1.2 \times 4190 \times 10^6}{10^3} = 502.8 \times 10^4 \text{ g}$$
Payload = weight of air displaced – (weight of balloon + weight of gas) Pay load
$$= 502.8 \times 10^4 - 78.13 \times 10^4$$
$$= 424.67 \times 10^4 \text{ g}$$

79. (a)
Since $P_{O_2} = X_{O_2} \times P_{total}$ then $X_{O_2} = \frac{P_{O_2}}{P_{total}}$
$$X_{O_2} = \frac{0.21}{8.38} = 0.025$$
So, the diving gas should contain 2.5% of O_2.

80. (a)
The rate of diffusion is directly proportional to the area of orifice.
$\therefore \quad d_A \ \alpha \ \pi r^2$
$d_B \ \alpha \ r^2$
$\therefore \quad \frac{d_A}{d_B} = \pi$
Hence, (A) is correct.

81. (a)
$$\frac{1}{2}mc^2 = \frac{3}{2}KT$$
Suppose the temperature required is T′ when the velocity will be $\frac{3}{2}$C
$$\frac{3/2C}{C} = \sqrt{\frac{T'}{T}} \text{ or, } T' = \frac{9}{4} \times 273 = 614.25 \text{ K}$$

82. (d)
P = 50 atm V = 350 ml = 0.350 litre n = 1 mole
T = 300K $\because Z = \frac{PV}{nRT}$
$$\therefore Z = \frac{50 \times 0.350}{1 \times 0.082 \times 300} = 0.711$$
Thus SO_2 is more compressible than expected from ideal behavior

83. (a)
$$\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$
At low pressures, 'b' can be ignored as the volume of the gas is very high. At high temperatures 'a' can be ignored as the pressure of the gas is high.
$\therefore \quad P(V-b) = RT$
$PV - Pb = RT \Rightarrow PV = RT + Pb$
$$\frac{PV}{RT} = Z = 1 + \frac{Pb}{RT}$$
Hence, (A) is correct.

84. (b)
Volume occupied by solute at NTP
= Volume of air displaced at NTP
= 112 ml
$\because$ For volatile solute $PV = \frac{w}{M}RT$
at NTP, P = 1 atm, T = 273 K
$$1 \times \frac{112}{1000} = \frac{0.23}{m} \times 0.0821 \times 273$$
m = 46.02 and V.D. = 23.01

85. (c)
Volume of 1 balloon which has to be filled
$$= \frac{4}{3}\pi\left(\frac{21}{2}\right)^3 = 4851 \text{ ml}$$
= 4.851 litre
Let n balloons be filled, then volume of H_2 occupied by balloons = 4.851 × n
Also, cylinder will not be empty and it will occupy volume of H_2 = 2.82 litre.
$\therefore$ Total volume occupied by H_2 at NTP = 4.851 × n + 2.82 litre
$\therefore$ At STP

$P_2 = 1$ atm | Available H_2
$V_1 = 4.851 \times n + 2.82$ | $P_2 = 20$ atm
$T_1 = 273$ K | $T_2 = 300$K

$\frac{P_1V_1}{T_2} = \frac{P_2V_2}{T_2}$ | $V_2 = 2.82$ litre

or $\frac{1\times(4.851n + 2.82)}{273} = \frac{20\times 2.82}{300}$ $\therefore$ n = 10

86. (a)
w = 20g dry CO_2 which will evaporate to develop pressure
m = 44, V = 0.75 litre, P = ? T = 298K

$PV = \frac{w}{m}RT$

$P \times 0.75 = \frac{20}{44} \times 0.0821 \times 298$

P = 14.828 atm
Pressure inside the bottle = P + atm pressure
= 14.828 + 1 = 15.83 atm

87. (b)
The water vapours occupies the volume of N_2 gas i.e. 50 litre
$\therefore$ For H_2O vapour V = 50 litre, w = 1.20g, T = 300K, m = 18

$PV = \frac{w}{m}RT$ or $P \times 50 = \frac{1.2}{18}\times 0.0821\times 300$

$\therefore$ P = 0.03284 atm
= 24.95 mm

88. (a)

$u_{av} = \sqrt{\frac{8RT}{\pi M}}$

Given $u_{av} = 0.3$ m sec^{-1} at 300K

$u_1 = 0.3 = \sqrt{\frac{8R\times 300}{\pi M}}$

at T = 273 + 927 = 1200K

$u_2 = \sqrt{\frac{8R\times 1200}{\pi M}}$

$\frac{u_2}{0.3} = \sqrt{\frac{1200}{300}}$ $u_2 = 0.6$ m sec^{-1}

89. (a)
For gaseous mixture 80% O_2, 20% gas
Average molecular weight M_m

$= \frac{32\times 80 + 20\times m}{100}$

Now, for diffusion of gaseous mixture and pure O_2

$\frac{r_{O_2}}{r_m} = \sqrt{\frac{M_m}{M_{O_2}}}$ or $\frac{V_{O_2}}{E_{O_2}}\times\frac{E_m}{V_m} = \sqrt{\frac{M_m}{M_{O_2}}}$

$\therefore \frac{1}{224}\times\frac{234}{1} = \sqrt{\frac{M_m}{32}}$ $\therefore M_m = 34.92$

$\therefore \frac{32\times 80 + 20\times m}{100} = 34.92$

$\therefore$ m = 46.6

90. (c)
For the cylinder V = constant
Hence $P_1V = n_1RT$ and $P_2V = n_2RT$

$\therefore \frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{\frac{w_1}{M}}{\frac{w_2}{M}} = \frac{w_1}{w_2}$

Hence $\frac{10}{8} = \frac{15}{w_2}$ $\therefore w_2 = 12$ kg

$\therefore$ Gas leaked out = 15 – 12 = 3 kg

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