A

TEST ID 00038784

HINTS AND SOLUTIONS

PAPER – I

CHEMISTRY

1. (A)
Let I_2 extracted into water = x g
I_2 left in CCl_4 = $(1 - x)$ g

$$\therefore\ C_{I_2}(CCl_4) = \frac{1-x}{10}\,g\,mL^{-1}$$

$$C_{I_2}(H_2O) = \frac{x}{400}\,g\,mL^{-1}$$

$$\frac{C_{I_2}(CCl_4)}{C_{I_2}(H_2O)} = 400 = \frac{\frac{1-x}{10}}{\frac{x}{400}}$$

$$\therefore\ x = \frac{1}{11}\,g$$

2. (C)
$H_2O(l) \rightleftharpoons H_2O(g)$. This is endothermic process, taking place with increase in pressure. If pressure is increased, equilibrium is displaced in backward side (Le-Chatelier) hence, steam is liquefied. To boil the liquid again, b.p. increases.

3. (C)

$$U_{rms}(H_2) = \sqrt{7}\,U_{rms}(N_2)$$

or $\sqrt{\dfrac{2r\,T_{H_2}}{2}} = \sqrt{7} \times \sqrt{\dfrac{3RT_{N_2}}{28}}$

$\therefore T_{N_2} = 2T_{H_2}$ or

$T_{N_2} > T_{H_2}$

4. (A)
70% by weight 70*gm* $H_3PO_4 \rightarrow$
100 *gm* solution / sample

$$V = \frac{W}{d} = \frac{100}{1.54}\,M = \frac{70 \times 1000}{98 \times 100/1.54} = 11M$$

Normality = 11 × 3 = 33N.

5. (B)
Mass of oxygen which gets displaced from metal oxide = 0.4 g
Now, 0.4 g of oxygen combines with metal = 3.2 g
8 g of oxygen combines with metal

$$\frac{3.2}{0.4} \times 8 = 64g$$

$\therefore$ GEW of metal = 64 g

Valency of metal = $\dfrac{GAM}{GEW} = \dfrac{64g}{64g} = 1$

Hence, formula of oxide is M_2O.

6. (A)
The acidic character follows the order: HOCl > HOBr > HOI on the basis of electronegativity of halogen

7. (C)
31 – 36 $\Rightarrow$ Ga to Kr.

8. (A), (B), (C), (D)
(A) The reaction is

$$BrO_3^- + 6H^+ + 6e^- \rightarrow Br^- + 3H_2O.$$

(B) The change in oxidation state of iodine in IO_3^- is from +5 to +1.

9. (A), (B), (C), (D)
(A) In the given reaction $Ca(OH)_2$ with two $2OH^-$ neutralizes $2H^+$ of H_3PO_4.
Thus, equivalent mass of H_3PO_4

$$= \frac{M}{2} = \frac{98}{2} = 49$$

(B) $CaHPO_4$ has one ionisable H^+

$$HPO_4^{2-} + OH^- \rightleftharpoons H_2O^+ + PO_4^{3-}$$

Thus, resulting mixture after first reaction is neutralized by 1 mol of KOH
(C) Since $CaHPO_4$ has one ionisable H^+ thus it is an acid salt
(D) $2H_3PO_4 + 3Ca(OH)_2 \rightarrow Ca_3(PO_4)_2 + 3H_2O$
$2H_3PO_4 \equiv 3Ca(OH)_2$
$1H_3PO_4 = 1.5\ Ca(OH)_2$

10. (A), (C)
The electronic configurations are as follows.
(a) ${}_{20}Ca\ (4s)^2$ and ${}_{19}K\ (4s)^1$. In calcium, electron is to be removed (i) fully-filled s orbital and (ii) against larger nuclear charge

11. (A), (B), (D)
In transition elements, the atomic radii becomes nearly equal beyond d^5 due to effective shielding of nuclear charge by (n – 1)d electrons

12. (D)

13. (C)

14. (B)
Rs. 50 per kg = Rs. 50 per $\left(\frac{1000}{74.5}\right)$mol

$\text{cost} = \frac{\text{Rs.}50}{\frac{1000}{74.5}}$ per mol = Rs. 3.73 per mol

15. (C)
1 kg KCl $= \frac{1000}{74.5}$ mol K^+ costing Rs. 50/-

Re 1 gives $= \frac{1000}{74.5 \times 50}$ mol of K^+ (in KCl)

$1\text{kg}\ K_2SO_4 = \frac{1000}{174}\text{mol} = \frac{2000}{174}$ mol

K^+ costing Rs.x

Re 1 gives $= \frac{1000 \times 2}{174x}$ mol of K^+ (in K_2SO_4)

$\therefore \frac{1000}{74.5 \times 50} = \frac{2000}{174x}$

X = Rs.42.82 kg^{-1}

16. (D)
Let mass of K_2O = xg

$= \frac{x}{94}$ mol K_2O

$= \frac{2x}{94}$ mol K^+

1000 g KCl = $\frac{1000}{74.5}\text{mol KCl} = \frac{1000}{74.5}\text{mol}\,K^+$

$\therefore \frac{2x}{94} = \frac{1000}{74.5}$

x = 631 g = 0.631 g

17. 1
$2Na_2S_2O_3 + I_2 \rightarrow Na_2\overset{+2.5}{S_4}O_6 + 2NaI$
n = 2 × 0.5 = 1

$E = \frac{M}{n - \text{factor}} = \frac{M}{1} = M$

18. 2
$\frac{r_g}{r_{He}} = \frac{\sqrt{M_{He}}}{M_g} \therefore M_g = M_{He}.\frac{r^2_{He}}{r^2_g}$

$= \frac{4}{(1.4)^2} = \frac{4}{1.96} = 2$

$\left[\text{Note}: 1.4 = \sqrt{2}\right]$

19. 4
O atoms in 2 g of oxygen
$= \frac{2 \times N_A}{16} = 0.125N_A$

4 g$\left(= \frac{4}{32}\text{g} - \text{atom}\right)$ of sulphur also contains atom = 0.125 N_A.

20. 4
$\frac{d_1}{d_2} = \frac{1}{16}; \frac{r_1}{r_2} = \sqrt{\frac{d_2}{d_1}} = \sqrt{16} = \frac{4}{1}$

21. 4
Molality
=
$\frac{\text{wt.of solute}}{\text{mol.wt.of solute} \times \text{wt.of solvent (g)}} \times 1000$

$= \frac{30}{30 \times 250} \times 1000 = 4$

22. 6
$6Fe^{2+} + Cr_2O_7^{2-} + 14H^+ \longrightarrow$
$6Fe^{3+} + 2Cr^{3+} + 7H_2O$

$\overset{+6}{Cr_2}O_7^{2-} \longrightarrow Cr^{3+}$
x factro = 6

Mohr's salt, $\overset{+2}{Fe}SO_4.(NH_4)_2SO_4.6H_2O$

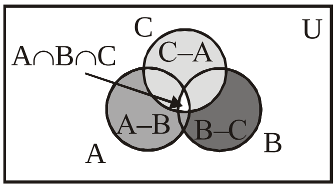
Oxidation $Fe^{2+} \longrightarrow Fe^{3+}$
x factor = 1
Mole ratio is reverse of x-factor ratio.
Thus, one mole of dichromate required = 6 moles of Mohr's salt

23. 2
The density of gas = $\frac{\text{molecular wt.of gas}}{\text{volume}}$

$= \frac{45}{22.4} = 2\text{gm litre}^{-1}$

MATHEMATICS

24. (C)
From Venn-Euler's Diagram,



Clearly, $\{(A - B) \cup (B - C) \cup (C - A)\}'$
$= A \cap B \cap C$.

25. (A)
$\because [\sin x] + \left[\sqrt{2}\cos x\right] = -3$

$\Rightarrow [\sin x] = -1$ and $\left[\sqrt{2}\cos x\right] = -2$

or $-1 \le \sin x < 0$ and $-2 \le \sqrt{2}\cos x < -1$

or $-1 \le \sin x < 0$ and $-\sqrt{2} < \cos x < -\frac{1}{\sqrt{2}}$

or $-1 \le \sin x < 0$ and $-1 \le \cos x < -\frac{1}{\sqrt{2}}$

$\therefore x \in (\pi, 2\pi)$ and $x \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$

$\therefore x \in (\pi, 2\pi) \cap \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$

$\therefore x \in \left(\pi, \frac{5\pi}{4}\right)$

26. (B)
Explanation: As there is no such number which is greater than 6 as well as less than 5. So $\{x : x < 5$ and $x > 6\}$ is a null set.
Also $|x|$ is always +ve so there exist no number which satisfy the condition $|x| < -2$
So $\{x : |x| < -2, x \in N\}$ is also a null set.

27. (A)
$x \ge 0 \Rightarrow |x| = x \Rightarrow |x| - x = 0,\ x - |x| = 0$
$\therefore$ Neither $f(x)$ nor $g(x)$ is defined.
$x < 0 \Rightarrow |x| = -x \Rightarrow |x| - x = -2x > 0$
And $x - |x| = 2x < 0$
$\Rightarrow \sqrt{|x| - x} > 0$ and $\sqrt{x - |x|}$ is not defined.
$\therefore f(x)$ is defined and $g(x)$ is not defined.
$\therefore D(f) = (-\infty, 0)$ and $D(g) = \phi$

28. (C)
We have $bN = \{bn : n \in N\}$,
$cN = \{cn : n \in N\}$
and $dN = \{dn : n \in N\}$
We have $bN \cap cN = dN$
$\therefore d = d.1 \in bN \cap CN$
$\Rightarrow d = bn_1$ and $d = cn_2$
where $n_1, n_2 \in N$
$\Rightarrow$ b/d and $\Rightarrow$ c/d $\Rightarrow$ bc/d, because *b* and *c* are coprime.
Also $bc \in bN$ and $bc = cb \in cN$
$\therefore bc \in bN \cap cN$ or $bc \in dN$
$\Rightarrow bc = dn_3$ for $n_3 \in N \Rightarrow$ d/bc
$\therefore bc = d$.

29. (B)
$[x]^2 = x + 2\{x\}$
$\Rightarrow [x]^2 = [x] + 3\{x\}$

$\Rightarrow \{x\} = \frac{[x]^2 - [x]}{3} < 1$

$\Rightarrow 0 \le \frac{[x]^2 - [x]}{3} < 1$

$\Rightarrow 0 \le [x]^2 - [x] < 3$

$\Rightarrow [x] \in \left(\frac{1-\sqrt{3}}{2}, 0\right] \cup \left[1, \frac{1+\sqrt{3}}{2}\right)$

$\Rightarrow [x] = -1, 0, 1, 2$

$\Rightarrow \{x\} = \frac{2}{3}, 0, 0, \frac{2}{3}$ (respectively)

$\Rightarrow x = -\frac{1}{3}, 0, 1, \frac{8}{3}$

30. (A)
$f(x) = (\sin^2\theta + \cos^2\theta + x)$

$$\begin{vmatrix} 1 & \cos^2\theta & x \\ 1 & x & \sin^2\theta \\ 1 & \sin^2\theta & \cos^2\theta \end{vmatrix}$$

$$= (x+1)\begin{vmatrix} 1 & \cos^2\theta & x \\ 0 & x - \cos^2\theta & \sin^2\theta - x \\ 0 & \sin^2\theta - \cos^2\theta & \cos^2\theta - x \end{vmatrix}$$

$= (x + 1)\,[(x - \cos^2\theta)(\cos^2\theta - x) -$

$(\sin^2\theta - \cos^2\theta)(\sin^2\theta - x)]$
$= (x+1)[-x^2 - \cos^4\theta + 2x\cos^2\theta$
$- x\cos^2\theta + x\sin^2\theta - \sin^4\theta + \sin^2\theta\cos^2\theta]$
$= (-1/2)(x+1)[(x-\sin^2\theta)^2 + (x-\cos^2\theta)^2 + (\sin^2\theta-\cos^2\theta)^2]$
So $f(x) = 0$ if $x = -1$ or $x = \sin^2\theta = \cos^2\theta$
$\sin^2\theta = \cos^2\theta \Rightarrow \theta = \pi/4 \Rightarrow x = 1/2$
Hence $x = -1, 1/2$

31. (A), (B), (C)

$\sqrt{\cos^2 x - \sin^2 x} + \sqrt{(\cos x + \sin x)^2}$
$= 2\sqrt{\cos x + \sin x}$
$\Rightarrow \sqrt{\cos x + \sin x}\left[\sqrt{\cos x - \sin x} + \sqrt{\cos x + \sin x}\right]$
$= 2\sqrt{\cos x + \sin x}$
$\Rightarrow$ either $\cos x + \sin x = 0 \Rightarrow \tan x = -1$
$\Rightarrow x = n\pi - \frac{\pi}{4}$ (n $\in$ I)
$\Rightarrow$ (A) and (C) are correct.
or $\sqrt{\cos x - \sin x} + \sqrt{\cos x + \sin x} = 2$
$\Rightarrow 2\cos x + 2\sqrt{\cos^2 x - \sin^2 x} = 4$
$\Rightarrow \cos^2 x - \sin^2 x = (2 - \cos x)^2 \Rightarrow \cos^2 x + 4\cos x - 5 = 0$
$\Rightarrow \cos x = \frac{-4 \pm \sqrt{16+20}}{2} = -5$ or 1
But $\cos x \neq -5$ so $\cos x = 1 \Rightarrow x = 2n\pi$
$\therefore$ (B) is also correct.

32. (B), (C)

33. (B), (C)

From Δ ABC , we get by the sine rule

$$\frac{AD}{\sin B} = \frac{BD}{\sin(A/2)} \quad \ldots(1)$$

Since the bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle. We have BD/DC = c/b. Also, BD + DC = BC = a.

Therefore,

$$BD = \frac{c}{b+c}.a \quad \ldots..(2)$$

From (1) and (2) we get

$$AD = \frac{ca}{b+c}.\frac{\sin B}{\sin(A/2)}$$

$$= \frac{2ca\sin B.\cos(A/2)}{(b+c)\sin(A/2).2\cos(A/2)}$$

$$= 2.\frac{c}{b+c}.\frac{a}{\sin A}.\sin B\cos\frac{A}{2}$$

$$2\frac{c}{b+c}.\frac{b}{\sin B}.\sin B\cos\frac{A}{2} = \frac{2bc\cos(A/2)}{b+c}$$

[by the law of sines]

Thus, (2) is correct

Also, AD = $\frac{2bc}{b+c}.\frac{\cos(A/2)\sin(A/2)}{\sin(A/2)}$

$$= \frac{bc\sin A}{(b+c)\sin(A/2)}$$

$$= \frac{abc}{2R(b+c)} \text{ cosec } \frac{A}{2}\left[\because \sin A = \frac{a}{2R}\right]$$

Therefore, (C) is also correct.

Again AD = $\frac{abc.4\Delta}{2abc(b+c)}$ cosec $\frac{A}{2}$

$$= \frac{2\Delta}{b+c} \text{ cosec } \frac{A}{2}\left[\because R = \frac{abc}{4\Delta}\right]$$

Hence (D) is not correct.

34. (A), (B), (C)

Let $f(x) = \cos nx \sin\left(\frac{5x}{n}\right)$

$\because f(x)$ is periodic $\Rightarrow f(x+\lambda) = f(x)$

$\cos(x+\lambda)\sin\left(\frac{5(x+\lambda)}{n}\right) = \cos nx \sin\frac{5x}{n}$

At $x = 0$, $\cos n\lambda \sin\frac{5\lambda}{n} = 0$

If $\cos x\lambda = 0$

$n\lambda = r\pi + \frac{\pi}{2}, r \in I$

$\because \lambda = 3\pi \Rightarrow$

$(3n - r) = \frac{1}{2}$ (which is impossible)

$\therefore \sin\frac{5\lambda}{n} = 0$

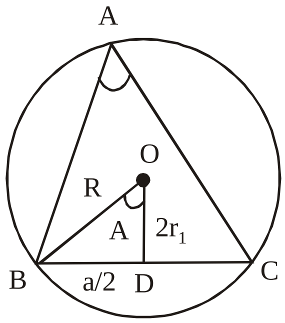
$\Rightarrow \frac{5\lambda}{n} = p\pi$, p $\in$ I

$\Rightarrow \frac{5(3\pi)}{n} = p\pi \Rightarrow n = \frac{15}{p}$

Now, p = ±1, ±3, ±5, ±15

$\therefore$ n = ±15, ±5, ±3, ±1

35. (A)
36. (B)
37. (B)



We know that Δ = abc/4R

so $\Delta_1 = \dfrac{OB \times OC \times BC}{4R_1} = \dfrac{aR^2}{4R_1}$

$\Rightarrow \dfrac{a}{R_1} = \dfrac{4\Delta_1}{R^2}$

similarly $\dfrac{b}{R_2} = \dfrac{4\Delta_2}{R^2}, \dfrac{c}{R_3} = \dfrac{4\Delta_3}{R^2}$

Hence $\dfrac{a}{R_1} + \dfrac{b}{R_2} + \dfrac{c}{R_3} = \dfrac{4}{R^2}(\Delta_1 + \Delta_2 + \Delta_3)$

$= \dfrac{4\Delta}{R^2}$

($\because \Delta = \Delta_1 + \Delta_2 + \Delta_3$)

38. (A)

$\angle BOD = \dfrac{1}{2}\angle BOC = A$

From Δ OBD, $\dfrac{a/2}{2r_1} = \tan A$

$\Rightarrow \dfrac{a}{r_1} = 4\tan A,$ similarly

$\dfrac{b}{r_2} = 4\tan B, \dfrac{c}{r_3}$ = 4 tan C

Hence $\dfrac{a}{r_1} + \dfrac{b}{r_2} + \dfrac{c}{r_3} = 4$

(tan A + tan B + tan C) = 4 tan A tan B tan C

39. (C)

$a = 4r_1 \tan A = \dfrac{4\Delta_1 R_1}{R^2}$

$\Rightarrow \dfrac{R_1}{r_1} = \dfrac{R^2 \tan A}{\Delta_1}$

similarly $\dfrac{R_2}{r_2} = \dfrac{R^2 \tan B}{\Delta_2}, \dfrac{R_3}{r_3} = \dfrac{R^2 \tan C}{\Delta_3}$

Hence

$\dfrac{R_1}{r_1} = \dfrac{R_2}{r_2} + \dfrac{R_3}{r_3} = R^2\left[\dfrac{\tan A}{\Delta_1} + \dfrac{\tan B}{\Delta_2} + \dfrac{\tan C}{\Delta_3}\right]$

40. 3

41. 2

The given equation can be written as $\dfrac{1+\sin x}{\cos x} = 2\cos x$

$\Rightarrow$ 1 + sin x = 2 cos^2 x = 2(1 – sin^2 x)

$\Rightarrow$ 2 sin^2 x + sin x = 0

$\Rightarrow$ (1 + sin x)(2 sin x – 1)= 0

$\Rightarrow$ sin x = –1 or 1/2

Now sin x = –1 $\Rightarrow$ x = 3π/2 for which the given equation is not meaningful.

and sin x = 1/2 $\Rightarrow$ x = π/6 or 5π/6

$\therefore$ The required number of solution is 2.

42. 2

2 cos 3B + 3 cos 4A = 3

$\Rightarrow$ 2 cos 3B = 3 (1 – cos 4A) = 6 sin^{2}2A

and 2 sin 3B – 3 sin 4A = 0

$\Rightarrow$2 sin 3B = 6 sin 2A cos 2A then

cot 3B = tan 2A

i.e. $\dfrac{\cos 3B}{\sin 3B} = \dfrac{\sin 2A}{\cos 2A}$ i.e. cos (2A + 3B) = 0

Since 0 < A, B < $\dfrac{\pi}{4}$ $\Rightarrow$ 0 < 2A + 3B < $\dfrac{3\pi}{4}$

$\therefore$ 2A + 3B = $\dfrac{\pi}{2}$

43. 2

sin α = $\dfrac{12}{37}$

cos β = $\dfrac{20}{101}$

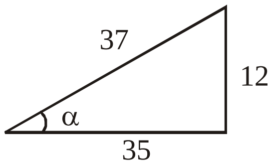
sin(α + β) = sin α cos β + cos α sin β

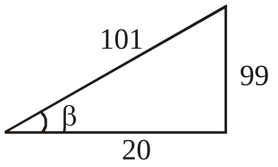
$= \dfrac{12}{37}.\dfrac{20}{101} + \left(-\dfrac{35}{37}\right)\left(-\dfrac{99}{101}\right)$

$= \dfrac{240 + (35)(99)}{(37)(101)} = \dfrac{3705}{3737}$

$\therefore$ cosec $(\alpha + \beta) = \dfrac{3737}{3705} = \dfrac{p}{q}$

$\Rightarrow (p + q) = 7442$





44. 1

A, B, C are in A.P. $\Rightarrow B = 60^o$

$\Rightarrow \cos B = \cos 60^o = \dfrac{1}{2} = \dfrac{c^2 + a^2 - b^2}{2ca}$

$\Rightarrow a^2 + c^2 = b^2 + ac$

$\Rightarrow (a - c)^2 = b^2 - ac$

$\Rightarrow |a - c| = \sqrt{b^2 - ac}$

$\Rightarrow |\sin A - \sin C| = \sqrt{\sin^2 B - \sin A \sin C}$

$\Rightarrow 2\cos\dfrac{A+C}{2}\left|\sin\dfrac{A-C}{2}\right| = \sqrt{\dfrac{3}{4} - \sin A \sin C}$

$\Rightarrow 2\left|\sin\dfrac{A-C}{2}\right| = \sqrt{3 - 4\sin A \sin C}$

So that

$$\lim_{A\to C}\frac{\sqrt{3-4\sin A\sin C}}{|A-C|} = \lim_{A\to C}\frac{2\sin\frac{A-C}{2}}{|A-C|} = 1$$

45. 5

$$f = \left(\frac{2\tan x}{1+\tan^2 x}\right) = \frac{2\cos^2 x(1+\tan^2 x + 2\tan x)}{2}$$

$= \cos^2(1 + \tan x)^2$

$= \dfrac{(1+\tan x)^2}{\sec^2 x} = \dfrac{(1+\tan x)^2}{1+\tan^2 x} = 1 + \dfrac{2\tan x}{1+\tan^2 x}$

$\Rightarrow f(x) = 1 + x$

$\Rightarrow f(4) = 5$

46. 5

PHYSICS

47. (D)

$R = \dfrac{V}{l} = \dfrac{8}{4} = 2$ ohm

$\dfrac{\text{DR}}{\text{R}} \times 100 = \dfrac{\text{DV}}{\text{V}} \times 100 + \dfrac{\text{DI}}{\text{I}} \times 100$

$= \dfrac{0.5}{8} \times 100 + \dfrac{0.2}{4} \times 100 = 11.25\%$

$\Rightarrow \quad R = (2 \pm 11.25)\Omega$

48. (D)

$[\eta] = [M L^{-1} T^{-2}]$

Hence, $\left[\sqrt{\dfrac{M}{\eta L}}\right] = \sqrt{\dfrac{[M]}{[M L^{-1} T^{-2}][L]}} = [T]$

49. (A)

$A_1 = 2,\ A_2 = 3, |\vec{A}_1 + \vec{A}_2| = 3$

$\Rightarrow |\vec{A}_1 + \vec{A}_2|^2 = 9 \Rightarrow A_1^2 + A_2^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 9$

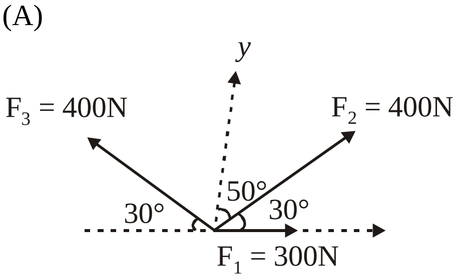
$\Rightarrow 2^2 + 3^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 9 \Rightarrow \vec{A}_1 \cdot \vec{A}_2 = -2$

Now,

$(\vec{A}_1 + 2\vec{A}_2)\cdot(3\vec{A}_1 - 4\vec{A}_2) = 3A_1^2 - 8A_2^2 + 2\vec{A}_1 \cdot \vec{A}_2$

$= 3(2)^2 - 8(3)^2 + 2(-2) = -64$

50. (A)



Net force along x-axis

$F_x = F_1 + F_2 \cos 30^o - F_3 \cos 30^o$

$F_x = 300$ N

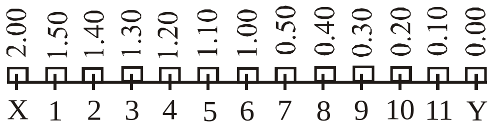
$F_y = F_2 \sin 30^o + F_3 \sin 30^o$

$= 400 \times \frac{1}{2} + 400 \times \frac{1}{2} = 400$ N

Net force: F = $\sqrt{300^2 + 400^2}$ = 500 N

51. (B)

Because one taxi leaves every 10 min, hence at any instant there will be 11 taxies on the way towards each station, one will be arriving and another leaving the other station. Figure shows the location of taxies going from X to Y at the instant 2.00 PM. The taxi which left station X at 0.00 PM has just arrived at station Y. Consider the taxi leaving the station Y at 2.00 PM.



It will meet all the 11 taxies marked 1 to 11 as well as 12 other taxies which would leave the station X from 2.00 PM to 3.50 PM. When it arrives at the station X at 4.00 PM, there will be one more taxi leaving that station. However, it will not be counted among the taxies crossed by taxi under consideration. That is, it will cross 23 taxies leaving the station X from 0.10 PM to 3.50 PM

52. (B)

Time taken by same ball to return to the hands of juggler $= \frac{2u}{g} = \frac{2 \times 20}{10} = 4s.$ So he is throwing the balls after each 1 s. Let a some instant he is throwing ball number 4. Before 1 s of it he throws ball 3. So height of ball 3 : $h_3 = 20 \times 1 - \frac{1}{2}10(1)^2 = 15$ m

Before 2 s, he throws ball 2. So height of ball 2:

$h_2 = 20 \times 2 - \frac{1}{2}10(2)^2 = 20$ m

Before 3 s, he throws ball 1. So height of ball 1:

$h_1 = 20 \times 3 - \frac{1}{2}10(3)^2 = 15$ m

53. (C)

$$\frac{ds}{dt} = 6t - \frac{t^2}{6}$$

Now on integrating both side $s = 3t^2 - \frac{t^3}{18} +$ constant, (where s is distance)

Now put $t = 0$, then $s = 0$ gives constant equal to 0 and putting t = 3, we get

$$s = 3(3)^2 - \frac{3^3}{18} = 27 - \frac{27}{18} = \frac{51}{2}.$$

54. (A), (B), (C)

Reynold's number and coefficient of friction are dimensionless quantities.

Curie is the number of atoms decaying per unit time and frequency is the number of oscillations per unit time.

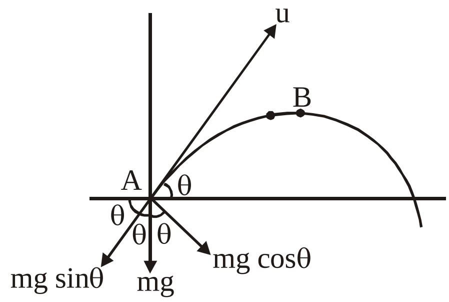
55. (A), (D)

At point A

$$a_C = \frac{V^2}{R}$$

$$g\cos\theta = \frac{u^2}{R}$$

$$\therefore\ R = \frac{u^2}{g\cos\theta}$$



At the highest point i.e., at point B

$$g = \frac{(u\cos\theta)^2}{R}$$

$$R = \frac{u^2\cos^2\theta}{g}$$

Choice (B) and (C) are wrong.

56. (B), (C), (D)

The value of any trigonometric function is a dimensionless number. Hence choice (A) is correct. The argument of a trigonometric function is also dimensionless. Hence (bt – cx) is dimensionless. Hence b has dimension $[T^{-1}]$ the same as that of frequency and c has dimension of $[L^{-1}]$. Thus choices (B), (C) and (D) are all correct.

57. (A), (C)

Since the initial velocity is zero, we have

$$h_1 = 0\times t_1 + \frac{1}{2}gt_1^2 = \frac{1}{2}gt_1^2$$

and $h_2 = 0\times t_2 + \frac{1}{2}gt_2^2 = \frac{1}{2}gt_2^2$

Therefore

$\frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$, which is choice (A).

Also, we have $v_1 = 0 + gt_1 = gt_1$ and $v_2 = gt_2$.

Therefore $\frac{v_1}{v_2} = \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$

which is choice (C)

Hence the correct choices are (A) and (C)

58. (D)

1 cal = 4.2 J = 4.2 kgm^2s^{-2}

$= n_2(\alpha\ \text{kg})\,(\beta m)^2\,(\gamma\, s)^{-2}$

$\Rightarrow n_2 = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$

So, 1 cal = $(4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2)$ new units.

59. (C)

Let $T \propto \sigma^a\ \rho^b\ r^c$

$M^0L^0T = (MT^{-2})^a\ (ML^{-3})^b\ L^c$

Equating the powers of

M : $0 = a + b \quad \Rightarrow b = -a$

L : $0 = -3b + c \Rightarrow c = 3b$

T : $1 = -2a \Rightarrow a = -\frac{1}{2},\ b = \frac{1}{2}, c = \frac{3}{2}$

$$\Rightarrow T = k\sqrt{\frac{r^3\rho}{\sigma}}$$

60. (B)

$y = ax - bx^2$

$$y = x\tan\theta - \frac{\alpha x^2}{2u^2\cos^2\theta}$$

$\tan\theta = a_1$ and $\frac{\alpha}{2u^2\cos^2\theta} = b$

$$u = \sqrt{\frac{a(1+a^2)}{2b}}\ .$$

61. (D)

$\frac{dx}{dt} = a$

$x = at$

$V_y = bx = b\,at$

$\frac{dy}{dt} = b\,at$

$\int dy = \int bat\ dt$

$y = \frac{bat^2}{2}$

$y = \frac{ba}{2}\frac{x^2}{a^2} = \frac{bx^2}{2a}$

$y \propto x^2$ i.e., parabolic.

62. (D)

Horizontal ranges are same for θ and (90 – θ), but maximum height is greater for larger angle.

63. 5

Relative density = $\frac{W_a}{W_a - W_w}$ or $\rho = \frac{W_a}{w}$

Where W_a weight in air and w is loss in weight.

$\frac{\Delta\rho}{\rho} = \frac{\Delta W}{W_a} - \frac{\Delta w}{w}$

For maximum error:

$\frac{\Delta\rho}{\rho} = \frac{\Delta W_a}{W_a} + \frac{\Delta w}{w}$

For maximum percentage error:

$\frac{\Delta\rho}{\rho}\times 100 = \frac{\Delta W_a}{W_a}\times 100 + \frac{\Delta w}{w}\times 100$

Given: ΔW_a = 0.1 gmf and W_a = 10.0 gmf

w = 10.0 – 5.0 = 5.0 gmf

$\Delta w = \Delta w_a + \Delta w_w$ = 0.1 + 0.1 = 0.2 gmf

$\therefore\ \frac{\Delta\rho}{\rho}\times 100 = \left(\frac{0.1}{10.0}\right)\times 100 + \left(\frac{0.2}{5.0}\right)\times 100$

= 1 + 4= 5%

64. 2

ℓ.c = 1m.s.d – 1 v.s.d

$= 1\text{ m.s.d.} - \frac{19}{20}\text{m.s.d.}\ [\because\ 20\ \text{v.s.d.} = 19\,\text{m.s.d.}]$

$= \frac{1}{20}$ m.s.d

$\therefore$ 1 m.s.d = 20 × ℓ.c = 20 × 0.1 = 2mm

[Alter: L.C. = $\frac{1}{n}$M.S.D

$\therefore$ 1 M.S.D = (n) L.C = 20 × 0.1 = 2 mm]

65. 9

Expanding P*v* – Pnb + $\frac{an^2}{v} - \frac{an^3b}{v^2}$

$= \left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{2}+1\right)+\left(2+\frac{1}{2}\right)+(3+1)=9\%$

66. 0

$T = 2\pi\sqrt{\frac{L}{g}}$ or $T^2 = 4\pi^2\frac{L}{g}$

$\Rightarrow g = 4\pi^2\frac{L}{T^2}; \frac{\Delta g}{g} = \frac{\Delta L}{L} - 2\frac{\Delta T}{T}$

$\frac{\Delta g}{g}\times 100 = \frac{\Delta L}{L}\times 100 - 2\frac{\Delta T}{T}\times 100$

Actual % error in g = $\frac{\Delta L}{L}\times 100 - 2\frac{\Delta T}{T}\times 100$

= + 2% – 2 × 1% = 0%

67. 2

$\vec{a}.\vec{b} = 0 \Rightarrow$ angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

68. 4

Let total distance = s

Let the time taken to cover first one third distance = t_1, then

$t_1 = \frac{s/3}{4} = \frac{s}{12}$

Now let t_2 be the time for the rest two journeys. Then

$\frac{2s}{3} = 2t_2 + 6t_2 = 8t_2$

$\therefore\ t_2 = \frac{2s}{24} = \frac{s}{12}$

$\therefore$ Average velocity $= \frac{\text{total displacement}}{\text{total time}}$

$= \frac{s}{t_1 + 2t_2} = \frac{s}{\frac{s}{12}+\frac{s}{6}} = \frac{12\times 6}{12+6} = 4\text{m/s}$.

69. 7

Area under acceleration-time graph gives change in velocity

Hence $A_{total} = \frac{4\times 4}{2} - 4\times 1$

= 8 – 4

$V_f - V_i = 4$

$V_f - 3 = 4$

$V_f = 7$ m/s.

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