B

TEST ID 00038785

HINTS AND SOLUTIONS

PAPER – II

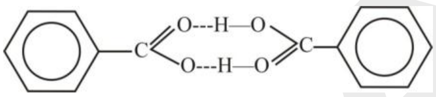
CHEMISTRY

1. (C)

$\dfrac{\text{conc. of benzoic acid in water}}{\text{conc. of benzoic acid in benzene}} = \dfrac{C_1}{C_2} \neq$ constant hence, monomer in both layer is not possible but $\dfrac{C_1}{\sqrt{C_2}}$ = constant.

I	II	III
$\dfrac{C_1}{\sqrt{C_2}} = 1,$	$\dfrac{C_1}{\sqrt{C_2}} = 1$	$\dfrac{C_1}{\sqrt{C_2}} = 1$

Thus, there is monomer in water and dimer in benzene. It is due to intermolecular H-bonding.



2. (B)

Work done = surface tension × increase in area

= 73 dyne cm^{-1} × 0.10 m^2

= 73 dyne cm^{-1} × 0.10 × 10^4 cm^2

= 7.3 × 10^4 erg

3. (C)

If w_n = solute extracted into solvent II by n extractions

w = solute initially in solvent I

V_1 = volume of solvent I

V_2 = volume of solvent II to be used in *n* parts

$\dfrac{C_2}{C_1} = k$ (Partition coefficient)

then $w_n = w - w\left(\dfrac{nV_1}{nV_1 + KV_2}\right)^n$

(I) If $n = 1, V_1 = 100\,mL,$

$V_2 = 100\,mL,$ w = 1 g, K = 2

then $w_1 = 1 - 1\left(\dfrac{1}{3}\right) = \dfrac{2}{3} = 0.667$

Thus, (A) is true

(II) If $n = 2, V_1 = 100\,mL,\ V_2 = 100\,mL,$

w = 1 g, K = 2

then $w_2 = 1 - 1\left(\dfrac{1}{2}\right)^2 = \dfrac{3}{4} = 0.75\,g$

thus, (B) is true also

4. (B)

If is about 0.224*L*.

5. (A)

Meq of $Mg^{+2} \equiv Meq$ of washing soda

$\dfrac{W}{E} \times 1000 = Mg^{+2};\ EW = \dfrac{24}{2} = 12$

$\dfrac{12 \times 10^{-3}}{12} \times 1000 = 1$

6. (B)

Cs > Rb > K > Na > Li

Metallic character decreasing order

7. (C)

Electron affinity increases with the decrease in the size and it also depends upon the electronic configuration

8. (B)

In the isoelectronic series all isoelectronic anions belong to the same period and cations to the next period.

9. (A), (B), (D)

The first ionization energy tends to increase along the period.

10. (A), (B), (C), (D)

11. (A), (B), (C)
The equation of an ideal gas are
pV = nRT
pV = nRT = (m/M)RT;
p = (m/V)RT/M = ρRT/M
$pV = (1/3)\, mN\, \overline{u^2}$

12. (A), (B), (C)
(A) $V_t = V_0 + \frac{t/^o C}{273} V_o$; intercept is not zero
(B) V_T = KT: intercept is zero
(C) V = KT; slope is K

13. 1
$uH_2 ; uO_2 = \sqrt{\frac{50}{2} \times \frac{32}{800}} = \sqrt{1} = 1$

14. 8
The reaction to be considered is
$4K_2Cr_2O_7 \rightarrow 4K_2CrO_4 + 2Cr_2O_3 + 3O_2$
The loss in mass is due to escape of O_2.
For 1 mol of $K_2Cr_2O_7$, (3/4) mol of O_2 is released. Hence

$$\frac{\text{mass of } (3/4) \text{mol of } O_2}{\text{mass of 1mol of } K_2Cr_2O_7} \times 100$$

$$= \frac{24\,g}{249\,g} \times 100 = 8\%$$

15. 1
$V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{1 \times 1 \times 250}{300 \times 0.75} = 1.11L$

16. 2
29.4 1b m^{-2} = $\frac{29.4}{14.7}$ atm. = 2atm.

17. 4

18. 6
$$\frac{P_1}{P_2} = \frac{d_1}{M_1}\frac{RTM_2}{d_2RT} = \frac{d_1M_2}{M_1d_2}$$
$$= \frac{P_x}{P_y} = \frac{d_xM_y}{d_yM_x} = \frac{3 \times 1}{0.5} = 6$$

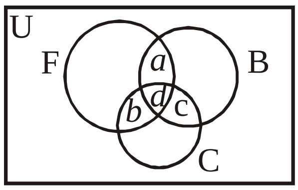
19. (A) – (r, t); (B) – (s); (C) – (p, s); (D) – (q)

20. (A) – (t); (B) – (q); (C) – (p); (D) – (r)

MATHEMATICS

21. (B)
tan *x* tan *y* = 1
or sin *x* sin *y* = cos *x* cos *y* or cos *x* cos *y* – sin *x* sin *y* = 0
or cos (*x* + *y*) = 0
⇒ *x* + *y* = π/2 but also *x* + *y* = 2.
Not possible.

22. (B)
Let F, B, and C denote the set of men who received medals in Football, Basketball and Cricket respectively. Then
n(F) = 38, n(B) = 15, n(C) = 20,
n(F ∪ B ∪ C) = 58 and n(F ∩ B ∩ C) = 3.
Therefore,
n(F ∪ B ∪ C)
= n(F) + n(B) + n(C) – n(F ∩ B)
–n(F ∩ C) – n(B ∩ C) + n(F ∩ B ∩ C).
Hence, n(F ∩ B) + n(F ∩ C) + n(B ∩ C)
= 18.
Consider the Venn diagram given in the figure



Here a denotes the number of men who got medals in Football and Basketball only, b denotes the number of men who got medals in Football and Cricket only, c denotes the number of men who got medals in Basketball and Cricket only and d denotes the number of men who got medals in all the three.
Thus, d = n(F ∩ B ∩ C) = 3
and a + d + b + c + d = 18

Therefore a + b + c = 9, which is the number of people who got medals in exactly two of the three sports.

23. (C)

24. (C)

$$f(x) = \log\left(\frac{1+x}{1-x}\right)$$

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right)$$

$= 2f(x)$

25. (D)

Given that $f(x) = (x+1)^2, x \geq -1$

Now if $g(x)$ is the reflection of $f(x)$ in the line $y = x$, then $g(x)$ is an inverse function of $y = f(x)$.

Given $(x+1)^2 (x \geq -1$ and $y \geq 0)$

$\Rightarrow x = \pm\sqrt{y} - 1$

$\Rightarrow g(x) = f^{-1}(x) = \sqrt{x} - 1, x \geq 0$

26. (B)

$m^2 + n^2 = 2 + 2(\cos A. \cos B + \sin A \sin B)$

$2mn = 2[\cos A \sin A + \cos A \sin B + \cos B \sin A + \cos B \sin B]$

$$\frac{m^2+n^2}{2mn} = \frac{2\{1+\cos(A-B)\}}{\sin 2A + \sin 2B + 2\sin(A+B)}$$

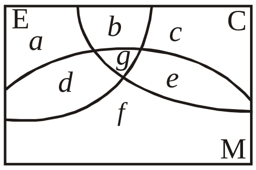
$$= \frac{2\{1+\cos(A-B)\}}{2\sin(A+B).\cos(A-B) + 2\sin(A+B)}$$

$$= \frac{2\{1+\cos(A-B)\}}{2\{1+\cos(A-B\}.\sin(A+B)}$$

$$\therefore \sin(A+B) = \frac{2mn}{m^2+n^2}$$

27. (A)

C stands for set of students taking economics



a + b + c + d + e + f + g = 40;
a + b + d + g = 16
b + c + e + g = 22; d + e + f + g = 26
b + g = 5; e + g = 14; g = 2
By backward substitution
e = 12, b = 3, d + f = 12, c + e = 17
$\Rightarrow$ c = 5; a + d = 11
a +d + f = 18 $\Rightarrow$f = 7
$\therefore$ d = 12 – 7 = 5

28. (A)

$$\Delta = \frac{1}{2}a\alpha = \frac{1}{2}b\beta = \frac{1}{2}c\gamma$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{a^2+b^2+c^2}{4\Delta^2}$$

and cot A + cot B + cot C

$$= \frac{2R}{2abc}[b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2]$$

$$\frac{R(a^2+b^2+c^2)}{abc} = \frac{a^2+b^2+c^2}{4\Delta}$$

Hence $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$

29. (C), (D)

$$y = \tan A.\tan\left(\frac{\pi}{3} - A\right)$$

$\Rightarrow \tan^2 A + \sqrt{3}(y-1)\tan A + y = 0$

As tan A is real, D $\geq$ 0 $\Rightarrow$ $3(y-1)^2 - 4y \geq 0$

$\therefore 3y^2 - 10y + 3 \geq 0 \Rightarrow y \leq \frac{1}{3}$ or $y \geq 3$.

But each of tan A, tan B is less than $\sqrt{3}$ or one is 0 and the other is $\sqrt{3}$.

$\therefore y \geq 3$ is not possible. So $y \leq \frac{1}{3}$.

Also, tan A. tan B $\geq$ 0

30. (A), (C), (D)

Let ABC be the isosceles triangle with AB = AC = b and $\angle$B = $\angle$C = α. Let AD be the perpendicular bisector of the side BC. Since ΔABC is isosceles, AD is also the bisector of angle A, so that O and I both lie on AD. We have OB = R and ID = r. Also, since from isosceles triangle OAB

$$\frac{OB}{\sin(90°-\alpha)} = \frac{AB}{\sin 2\alpha}$$

$$\Rightarrow R = \frac{b\cos\alpha}{2\sin\alpha\cos\alpha} = \frac{1}{2}b\,\text{cosec}\,\alpha$$

so that (A) is correct.

Again Δ = BD. AD = b cos α. b sin α = $\frac{1}{2}b^2\sin 2\alpha$

so that (B) is not correct.

Also, r =

$$\frac{\Delta}{s} = \frac{\frac{1}{2}b^2\sin 2\alpha}{\frac{1}{2}(b+b+2b\cos\alpha)} = \frac{b\sin 2\alpha}{2(1+\cos\alpha)}$$

so that (C) is correct.

Further OI = |OD + DI|= |OD + r|

Because α < π/4, A > π/2 and O lies on AD produced.

Now, from right-angled triangle ODB, we get

$OD^2 = OB^2 - BD^2 = R^2 - (b\cos\alpha)^2$

$$= \frac{1}{4}\frac{b^2}{\sin^2\alpha} - b^2\cos^2\alpha$$

$$= \frac{b^2(1-4\sin^2\alpha\cos^2\alpha)}{4\sin^2\alpha} \text{ [from (A)]}$$

$$= \frac{b^2(\cos^2\alpha-\sin^2\alpha)^2}{4\sin^2\alpha} = \frac{b^2\cos^2 2\alpha}{(2\sin\alpha)^2}$$

$$\Rightarrow OD = \frac{b\cos 2\alpha}{2\sin\alpha}$$

$$\therefore OI = \left|\frac{b\sin 2\alpha}{2(1+\cos\alpha)} + \frac{b\cos 2\alpha}{2\sin\alpha}\right|$$

$$= \left|\frac{b\sin 2\alpha}{4\cos^2(\alpha/2)} + \frac{b\cos 2\alpha}{4\sin(\alpha/2)\cos(\alpha/2)}\right|$$

$$= \left|\frac{b}{4\cos(\alpha/2)}.\frac{\sin 2\alpha\sin(\alpha/2)+\cos 2\alpha\cos(\alpha/2)}{\sin(\alpha/2)\cos(\alpha/2)}\right|$$

$$= \left|\frac{b\cos(3\alpha/2)}{2\sin\alpha\cos(\alpha/2)}\right|$$

Thus, (D) is also correct

31. (A), (C)

$$-\sqrt{a^2+b^2} \le a\cos\theta \pm b\sin\theta \le \sqrt{a^2+b^2}$$

$$-\sqrt{6^2+8^2} \le 6\cos\theta - 8\sin\theta \le \sqrt{6^2+8^2}$$

$$-10 \le 6\cos\theta - 8\sin\theta \le 10$$

$$-6 \le 6\cos\theta - 8\sin\theta + 4 \le 14$$

32. (A), (D)

$$\Delta = \frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3$$

$$\Rightarrow p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$$

p_1, p_2, p_3 are in H.P.

Now

$$p_1 = \frac{2\Delta}{2R\sin A}, p_2 = \frac{2\Delta}{2R\sin B}, p_3 = \frac{2\Delta}{2R\sin C}$$

$$\Rightarrow \frac{1}{p_1} = \frac{R\sin A}{\Delta}, \frac{1}{p_2} = \frac{R\sin B}{\Delta}, \frac{1}{p_3} = \frac{R\sin c}{\Delta}$$

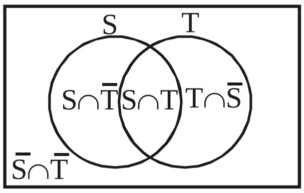
⇒ sin A, sin B, sin C are in A.P.

⇒ sin A + sin C = 2 sin B

$$\therefore \frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3} = \frac{R}{\Delta}(\sin A + \sin B + \sin C)$$

$$= \frac{3R}{\Delta}\sin B \le \frac{3R}{\Delta}.$$

33. 7



n(S) = 5 + 10 + ... + 1000 = 200 – 1 = 199

n(T) = 7 + 14 + ... + 994 = 142

n(S ∩ T) = 35 + 70 + ... + 980 = 28

n(S ∪ T) = 341 – 28 = 313

= n(S) + n(T) – n(S ∩T)

Total = 999

$n(\bar{S} \cap \bar{T})$ = 999 – 313 = 686

34. 0

Σcos 3x = 0

⇒Σ (4 $\cos^3 x$ – 3 cos x) = 0

⇒Σ $\cos^3 x$ = 0

$\Rightarrow \cos x \cos y \cos z = 0$

[using $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$]

Assume if cos z = 0,

$\Rightarrow \cos x = -\cos y$

$\Rightarrow \cos 2x = \cos 2y$ and $\cos 2z = -1$

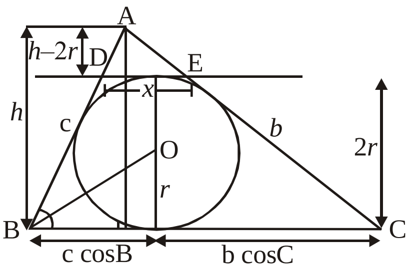
$\therefore \Pi \cos 2x = -\cos^2 2x \le 0$

$\therefore$ maximum value of $\Pi \cos 2x = 0$.

35. 8

36. 1

As ΔADE and ABC are similiar.



$$\Rightarrow \frac{x}{a} = \frac{h-2r}{h} = 1 - \frac{2r}{h}\left(r = \frac{\Delta}{s}\right)$$

$$= 1 - \frac{2\Delta}{sh} = 1 - \frac{ah}{sh} = 1 - \frac{a}{s}$$

$$\therefore \sum \frac{x}{a} = 3 - \left(\frac{a+b+c}{s}\right) = 3 - 2 = 1$$

$= 3 - 2 = 1$

37. 8

$x^2 + y^2 \le 144$ and $\sin(x + y) \ge 0$

$\Rightarrow 2n\pi \le x + y \le (2n + 1)\pi$; $n \in I$

Hence we get the area

$$S = \frac{\pi 144}{2} \Rightarrow \frac{S}{\pi} = 72, \frac{S}{9\pi} = 8$$

38. 2

For third side $(x - \sqrt{2})^2(x^2 + 1) \le 0$

$\Rightarrow (x - \sqrt{2})^2 \le 0 \Rightarrow x = \sqrt{2}$

$\therefore$ Third side must be $\sqrt{2}$

Clearly it is the greatest side, which is given by a

$$\therefore \cos A = \frac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 - 2}{2(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)} = 0$$

$$\Rightarrow A = \frac{\pi}{2}$$

39. (A) – (s); (B) – (p); (C) – (r); (D) – (t)

40. (A) – (q); (B) – (s); (C) – (p); (D) – (r)

(A) $5 \sin\theta + 3(\sin\theta \cos\alpha - \cos\theta \sin\alpha)$

$= (5 + 3\cos\alpha)\sin\theta - 3\sin\alpha \cos\theta$

$\Rightarrow \max\limits_{\theta \in R} \{5\sin\theta + 3\sin(\theta - \alpha)\}$

$= \sqrt{(5 + 3\cos\alpha)^2 + 9\sin^2\alpha}$

$= \sqrt{34 + 30\cos\alpha}$

$\therefore$ the given equation is $34 + 30\cos\alpha = 49$.

$\Rightarrow \cos\alpha\ 1/2 \Rightarrow \alpha = 2n\pi \pm \frac{\pi}{3}$, $n \in Z$

(B) $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$

$\Rightarrow \sin^2 x - 3\sin x + 2 = 0$

$\Rightarrow (\sin x - 2)(\sin x - 1) = 0 \Rightarrow \sin x = 1$

but this does not satisfy the equation because $0^0 = 1$ is absurd.

(C) $\sqrt{\sin x} + 2^{1/4}\cos x = 0$

$\therefore \sqrt{(\sin x)} > 0$ and so $\cos x < 0$,

i.e., x lies in 2nd quadrant or 3rd quadrant.

Equation (1) can be rewritten as

$2^{1/4}\cos x = -\sqrt{(\sin x)}$,

squaring $\sqrt{2}\cos^2 x = \sin x$

$\Rightarrow \sqrt{2}\sin^2 x + \sin x - \sqrt{2} = 0$

$\Rightarrow (\sqrt{2}\sin x - 1)(\sin x + \sqrt{2}) = 0$

$\sin x \ne -\sqrt{2}$ $\therefore \sin x = 1/\sqrt{2}$

Form equation (2) $\cos x < 0$ so $x = 3\pi/4 \in$ 2nd quadrant.

and so the general value of x is given by

$x = 2n\pi + 3\pi/4$, $n \in Z$

(D) $\log_5 \tan x = (\log_5 4)(\log_4(3\sin x))$

$\Rightarrow \log_5 \tan x = \log_5 (3 \sin x)$

Since log x is real when $x > 0$,

We have $\tan x > 0$, $\sin x > 0$

$\therefore$ x lies in first quadrant. Now equation (1) gives $\tan x = 3\sin x$ or $\cos x = 1/3$

$\therefore x = 2n\pi + \cos^{-1}(1/3)$

PHYSICS

41. (A)

Given that rate of metal increasing

$= 4\text{cm/sec} = v = \dfrac{da}{dt}$

We know that area of square of square sheet (A) = a^2, (where a is side).

$\therefore \dfrac{dA}{dt} = 2a\dfrac{da}{dt} = 2\times2\times4 = 16$ cm²/sec.

42. (A)

Let $\sqrt{(x^2-a^2)} = t$

$\Rightarrow x^2 - a^2 = t^2 \Rightarrow x^2 = a^2 + t^2$

$\therefore xdx = tdt$

$\therefore \displaystyle\int \frac{\sqrt{(x^2-a^2)}}{x}dx = \int \frac{\sqrt{(x^2-a^2)}x}{x^2}dx$

$\Rightarrow I = \displaystyle\int \frac{t}{a^2+t^2}tdt = \int \frac{t^2}{a^2+t^2}dt$

$\Rightarrow I = \displaystyle\int\left(1-\frac{a^2}{a^2+t^2}\right)dt = t - a^2\frac{1}{a}\tan^{-1}\left(\frac{1}{a}\right)$

$\Rightarrow I = \sqrt{(x^2-a^2)} - a\tan^{-1}\left[\dfrac{\left\{\sqrt{(x^2-a^2)}\right\}}{a}\right].$

43. (D)

Range = 150 = ut and $\text{h} = \dfrac{15}{100} = \dfrac{1}{2}\times \text{gt}^2$

or $\text{t}^2 = \dfrac{2\times15}{100\times\text{g}} = \dfrac{30}{1000}$ or $\text{t} = \dfrac{\sqrt{3}}{10}$.

$\text{u} = \dfrac{150}{\text{t}} = \dfrac{150\times10}{\sqrt{3}} = 500\sqrt{3}\ \text{ms}^{-1}$.

44. (A)

Let $Y = [V^aA^bF^c]$

$[ML^{-1}T^{-2}] = [LT^{-1}]^a[LT^{-2}]^b[MLT^{-2}]^c$

$ML^{-1}T^{-2} = M^cL^{a+b+c}T^{-a-2b-2c}$

$\therefore c = 1, a+b+c = -1, -a-2b-2c = -2$

On solving, we get $a = -4$, $b = 2$ and $c = 1$

45. (D)

$\left[\dfrac{mr^2}{6\pi\eta}\right] = \left[\dfrac{ML^2}{ML^{-1}T^{-1}}\right] = \left[L^3T\right]$

As we have $[\eta] = [ML^{-1}T^{-1}]$

$\left[\left(\dfrac{6\pi mr\eta}{g^2}\right)^{\frac{1}{2}}\right] = \left[\left(\dfrac{M\,L\,M\,L^{-1}T^{-1}}{L^2T^{-4}}\right)^{\frac{1}{2}}\right]$

$\left[\dfrac{m}{6\pi\eta rv}\right] = \left[\dfrac{M}{ML^{-1}T^{-1}LLT^{-1}}\right] = \left[L^{-1}T^{-2}\right]$

Thus, none of the given expressions have the dimensions of time.

46. (B)

Dimensions of

$P^xQ^yc^z = \left[ML^{-1}T^{-2}\right]^x\left[MT^{-3}\right]^y\left[LT^{-1}\right]^z$

As it is dimensionless, so

$\left[ML^{-1}T^{-2}\right]^x\left[MT^{-3}\right]^y\left[LT^{-1}\right]^z = \left[M^0L^0T^0\right]$

or $\left[M^{x+y}L^{-x+z}T^{-2x-3y-z}\right] = \left[M^0L^0T^0\right]$

Comparing powers of M, L and T, we get $x+y=0, -x+z=0, -2x-3y-z=0$

Solving, $x=1, y=-1, z=1$.

47. (C)

Let the man starts crossing the road at an angle θ with the road side. For safe crossing, the condition is that the man must cross the road by the time truck describe the distance (4 + 2 cotθ)

So, $\dfrac{4+2\cot\theta}{8} = \dfrac{2I\sin\theta}{V}$

or $V = \dfrac{8}{2\sin\theta+\cos\theta}$

For minimum V, $\dfrac{dv}{d\theta} = 0$

or $\dfrac{-8(2\cos\theta-\sin\theta)}{(2\sin\theta+\cos\theta)^2} = 0$

or $2v\cos\theta - \sin\theta = 0$

or $\tan\theta = 2$, so $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

$$V_{min} = \frac{8}{2\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{\sqrt{5}}} = \frac{8}{\sqrt{5}} = 3.57\,\text{m/s}$$

48. (C)

$y = ax - bx^2$, for height or y to be maximum: $\frac{dy}{dx} = 0$ or $a - 2bx = 0$

or $x = \frac{a}{2b}$

(i) $y_{max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$

(ii) $\left(\frac{dy}{dx}\right)_{x=0} = a = \tan\theta_0$ where θ_0

= angle of projection $\theta_0 = \tan^{-1} a$.

49. (A), (B)

Component of $\vec{A}$ along $\vec{B}$ is $|\vec{A}|\cos\theta\hat{B}$ for θ being the angle between the vectors.

Also $\hat{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$. So, choice (A) is correct.

The vector $(\hat{i} - \hat{j})$ is perpendicular to the vector $(\hat{i} + \hat{j})$. So, the other resolved component is $|\vec{A}|\sin\theta\left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right)$.

50. (A), (B), (D)

For the same range $\alpha + \beta = 90°$ or $\beta = 90° - \alpha$. Choice (A), (B) and (D) satisfy this relation between β and α but choice (C) does not.

51. (A), (B), (C)

Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}(1 - e^{-bt})\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}(ae^{-bt}) = -abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}\left(1 - e^{-1}\right);\ \frac{a}{b}\left(1 - \frac{1}{3}\right);\ \frac{2}{3}\frac{a}{b}\left(\because e^{-1};\ \frac{1}{3}\right)$$

Hence choice (A) is correct. At $t = 0$, the values v and α respectively are $v = ae^{-0} = a$ and $\alpha = -abe^{-0} = -ab$. Hence choice (B) is also correct. The displacement x is maximum when $t \to \infty$, i.e.

$x_{max} = \frac{a}{b}\left(1 - e^{-\infty}\right) = \frac{a}{b}$. Hence choice (C) is also correct.

52. (A), (B), (C)

$$\text{Let } I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)}\,dx \qquad (1)$$

$$= \int_\alpha^\beta \frac{f(a+b-x)}{f(a+b-x) + f(x)}\,dx \qquad (2)$$

Adding equations (1) and (2), we get

$\Rightarrow 2I = \int_a^b 1\,dx = b - a$

$\Rightarrow I = I = \left(\frac{b-a}{2}\right) = 10$ (given)

$\therefore\ b - a = 20$

53. 9

Motion of a particle $s = 15t - 2t^2$

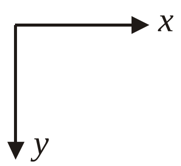
Therefore, velocity $\frac{ds}{dt} = 15 - 4t$

$\Rightarrow \left(\frac{ds}{dt}\right)_{t=0} = 15$ and $\left(\frac{ds}{dt}\right)_{t=3} = 3$

Therefore, average = $\frac{15 + 3}{2} = 9$

54. 4

Here we can take point of dropping as origin and downward direction as positive. Given, h = + 40 m, u = – 10 ms^{-1}, a = g = + 10 ms^{-2}



The initial velocity of the bag will be same as that of the velocity of balloon. As it moves in opposite direction of the balloon, so it will have negative sign with it. As we have values of h, u and g, so time can be obtained by the distance formula.

$\therefore$ formula used : $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ (where symbol have their usual meaning).

So $40 = -10t + \frac{1}{2}10t^2 \Rightarrow 5t^2 - 10t - 40 = 0$

$\Rightarrow 5t^2 - 20t + 10t - 40 = 0$

$5t(t-4) + 10(t-4) = 0$

$\Rightarrow (t-4)(5t+10) = 0$

$\Rightarrow t = 4s$ and – 2s

As time cannot be negative, so time taken by body to reach the ground is t = 4s.

55. 1

$\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = (2t-4)\hat{i} + 2t\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$

If $\vec{a}$ and $\vec{v}$ are perpendicular $\vec{a}.\vec{v} = 0$

$(2\hat{i} + 2\hat{j}).((2t-4)\hat{i} + 2t\hat{j}) = 0$

8t – 8 = 0

T = 1 sec.

56. 6

Heat produced H is given by

$$H = \frac{I^2Rt}{J}$$

$$\therefore \frac{\Delta H}{H} = 2\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t} - \frac{\Delta J}{J}$$

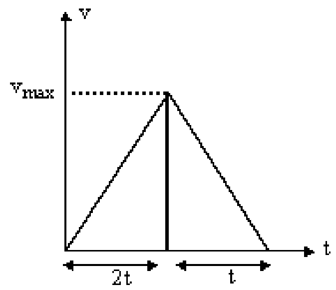
$$\frac{\Delta H}{H}\times 100 = 2\left(\frac{\Delta I}{I}\times 100\right) + \frac{\Delta R}{R}\times 100 + \frac{\Delta t}{t}\times 100 - \frac{\Delta J}{J}\times 100$$

For maximum percentage error.

$$\frac{\Delta H}{H}\times 100 = 2\frac{\Delta I}{I}\times 100 + \frac{\Delta R}{R}\times 100 + \frac{\Delta t}{t}\times 100 + \frac{\Delta J}{J}\times 100$$

= 2 × 2% + 1% + 1% + 0% = 6%.

57. 2



$\frac{v_{max}}{2t} = 10$

$\therefore v_{max} = 20t$

Area of graph

$\frac{1}{2}v_{max}\times 3t = 200 \Rightarrow \frac{3}{2}t\times 20t = 200$

$\Rightarrow t = \sqrt{\frac{20}{3}}$

Total time = 3t = $2\sqrt{15}$ sec

58. 4

Time of crossing = $\frac{800}{4} = 200s$,

Drift of the boat = [4 + (–2)] × 200 = 400m

59. (A) – (p, q); (B) – (p, q); (C) – (r); (D) – (p, q)

With constant positive acceleration, speed will increase when velocity is positive, speed will decrease if velocity is negative.

Similarly, with constant negative acceleration speed will increase if velocity is also negative and speed will decrease if velocity is positive.

60. (A) – (p); (B) – (p); (C) – (s); (D) – (r)

all digits are significant after decimal

$47.23 \div (2.3) = 20.5 \approx 21$ (should have only two significant digits)

3 is a number with one significant digit.